

# Kinetic-based CFD modeling of synchrotron emission spectra at fast SNRs

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**Abstract:** Recent kinetic particle-in-cell (PIC) simulations show that high-Mach number collisionless shocks can produce steep electron spectra. Due to short large-amplitude magnetic structures (SLAMS) driven in the upstream region, cosmic-ray (CR) electrons accelerate by a mechanism that is distinctly different from diffusive shock acceleration (DSA). Here we use the insights from the PIC runs together with optical observations to model the radio-synchrotron spectra observed at young and fast supernova remnants (SNRs), which show slopes similar to those from the PIC runs. First, we perform computational fluid dynamics (CFD) simulations of SNR shocks propagating in the homogeneous and clumpy interstellar media. To exclude spectral steepening due to non-linear DSA, we assume unmodified shocks. We then apply our kinetic-based semi-analytical model of particle acceleration with SLAMS to calculate the magnetic fields and CR electron spectra from the fluid velocity obtained at each computational cell of the detected shock filaments. We also introduce a concept of maximum acceleration energy for electrons that depends on the shock Mach number and the strength of the ambient field. Finally, we discuss the influence of inhomogeneities and CR-driven bubbles on the shape and slope of the synchrotron spectra throughout the adiabatic stage of SNR evolution.

## INTRODUCTION

During the early stages of evolution of the supernova remnant (SNR), a shock wave which is pushed by the ejecta can reach very high velocities  $V_{sh} \sim 10000 - 40000$  km/s (see e.g., Bell et al. 2011). Energetic cosmic rays (CRs) accelerate by scattering from the self-driven instabilities (Amato & Blasi 2009) in the upstream and their compressed counterparts in the downstream of the shock wave. This mechanism is known as a diffusive shock acceleration (DSA; Axford 1977, Bell 1978, Blandford & Ostriker 1978). These CR-driven instabilities excite circularly polarized resonant and non-resonant waves (also known as Bell modes; Bell 2004). Recent kinetic shock simulations (e.g., Caprioli & Spitkovsky 2014, Park et al. 2015) reveal what plasma processes drive the DSA and lead to the formation of the observed power-law spectra of ions and electrons.

It is shown by particle-in-cell (PIC) simulations (Zeković et al. 2024) that the CR-driven waves can turn into short large-amplitude magnetic structures (SLAMS) in the non-linear stage of their evolution. SLAMS are commonly observed as a series of quasi-periodic magnetic pulses ahead of the low-Mach number Earth bow shock (Willson et al. 2013). At high-Mach number astrophysical shocks SLAMS propagate with the super-Alfvénic velocity through the upstream and thus drive shocklets which quasi-periodically reform the primary shock. At fast quasi-parallel shocks shocks with  $V_{sh} > 10000$  km/s, SLAMS can become superluminal for magnetized electrons due to a large magnetic field amplification ( $> 30$ ). Such a large amplification and quasi-periodicity of the SLAMS magnetic field open a window for fast electron acceleration by the mechanism called *quasi-periodic shock acceleration* (QSA; Zeković et al. 2024). The QSA mechanism which leads to steep electron spectra with  $f(p) \sim p^{-(5-6)}$  shows the kinetic picture of the known macroscopic model of oblique shocks (Bell et al. 2011). In this macroscopic approach, it is argued that higher order anisotropies which appear at oblique high-Mach number shocks of young SNRs are able to produce steep electron spectra. The derived electron distribution  $f(p) \sim p^{-5}$  then implies the synchrotron emissivity  $\epsilon_\nu \sim \nu^{-6}$  with the spectral index  $\alpha \sim 1$ , which is steeper than the DSA prediction  $f(p) \sim p^{-4}$  and  $\epsilon_\nu \sim \nu^{-0.5}$  but well agrees with the spectra of electrons accelerated by QSA.

## MODELING ELECTRON SYNCHROTRON SPECTRA AT YOUNG SNRS

We model the electron synchrotron spectra in the cases of uniform and clumpy interstellar medium (ISM) by using the hydrodynamics code of Kostić (2019). Supernova remnants (SNRs) are simulated in a form of octants in 4 pc cubic box with the resolution  $540^3$  cells. The SNR is initiated by ejecta that has the density and velocity profiles with integrated mass and energy of  $1.4 M_{sun}$  and  $10^{51}$  ergs, respectively. Starting with the initial ejecta radius of 0.4 pc at  $t=0$ , the SNR expands to the radius of 4 pc in  $\sim 250$  years. Two SNR simulations are carried out: 1) in the uniform medium, and 2) in the clumpy medium. In the uniform case, the ambient density is  $0.05 \text{ cm}^{-3}$  (i.e. the concentration of H atoms). In the second simulation shown in Fig. 1, spherical clumps of 0.3 pc in diameter are randomly distributed beyond the radius of 2 pc (from the explosion), with a volume filling of 10%. The clumps density is 10 times higher ( $\rho_{clump} = 10 \rho_0$ ) than the surrounding uniform ISM which has the same density as in the first run. Magnetic flux is frozen inside the clumps and field is amplified as  $B_{clump} \sim B_0 \cdot (\rho/\rho_0)^{2/3}$ . The code uses shock detection algorithm to find the forward shock and its local velocity.

We introduce two acceleration regimes which we use to calculate synchrotron emission for each shock cell in the simulation.

**1. QSA regime** – which operates at high Mach numbers, when the shock is fast enough so that the condition  $V_{sh} \cdot B/B_0 > c$  for superluminal SLAMS is fulfilled. For the electron spectra in this case we infer the QSA slope  $\delta_e = -5$  from Zeković et al. (2024) and Hemler et al. (2024). The maximum momentum that CR electron can reach by QSA is set by the resonant condition. When electron Larmor radius becomes comparable to the scale of SLAMS, electrons become unmagnetized and exit QSA.

**2. DSA regime** – at lower Mach numbers when SLAMS are subluminal we use the DSA slope  $\delta_e = 4$ . The DSA regime appears only when shock enters the clumps. We first find the time required for the shock to sweep one whole clump as  $\delta t = 0.3 \text{ pc} / V_{sh}$ . We then use  $\delta t$  in the solution of the time dependent diffusion-advection equation (Drury 1983) to calculate the maximum momentum that CR electrons can reach after the time  $\delta t$ .

Once the shock encounters the clumps that are seeded in the upstream, both shock speed and sound speed drop by a factor of  $10^{0.5}$  keeping the sonic Mach number unchanged. However, the Alfvén Mach number decreases by  $10^{2/3}$ . The amplification of the magnetic field is set by the Alfvén Mach number through the relation which accounts for both resonant and non-resonant, CR-driven, upstream waves. The change in Alfvénic Mach number thus sets a switch between QSA and DSA. For each shock cell in the simulation we check this condition and calculate the synchrotron emission spectra accordingly. For this we use the analytical relation found in Pacholczyk (1970) with the spectral slopes and terminal energies from QSA or DSA. We sum the spectra from all shock cells and show the integral spectra in Fig. 2 at the different stages of evolution of the young SNR.

## RESULTS

### Double spectra at young SNRs

The steep synchrotron spectrum with the spectral index  $\alpha \approx 1$  is expected from electrons which accelerate by the QSA mechanism (Zeković et al. 2024). We find that such a steep slope is maintained in the electron synchrotron spectra during the whole evolution of young SNR. Throughout the run we observe that electrons accelerate to  $\sim \text{GeV}$  energies by the QSA mechanism. We thus observe that the electron spectra brake at  $\sim 1$  GHz frequency (see Fig. 2). In the case of uniform ISM, the spectra continues to fall off after the brake, while in the case with clumps the spectra flattens to the spectral index  $\alpha \approx 0.5$  at frequencies  $\sim 20\text{-}30$  GHz. This results in the two overlapping spectra which have their origin in the two different acceleration regimes – QSA and DSA. Therefore, our model provides an alternative explanation for the double spectra which are typically seen at Novae (e.g., in the case of recurrent Nova RS Ophiuchi; Diesing et al. 2023).

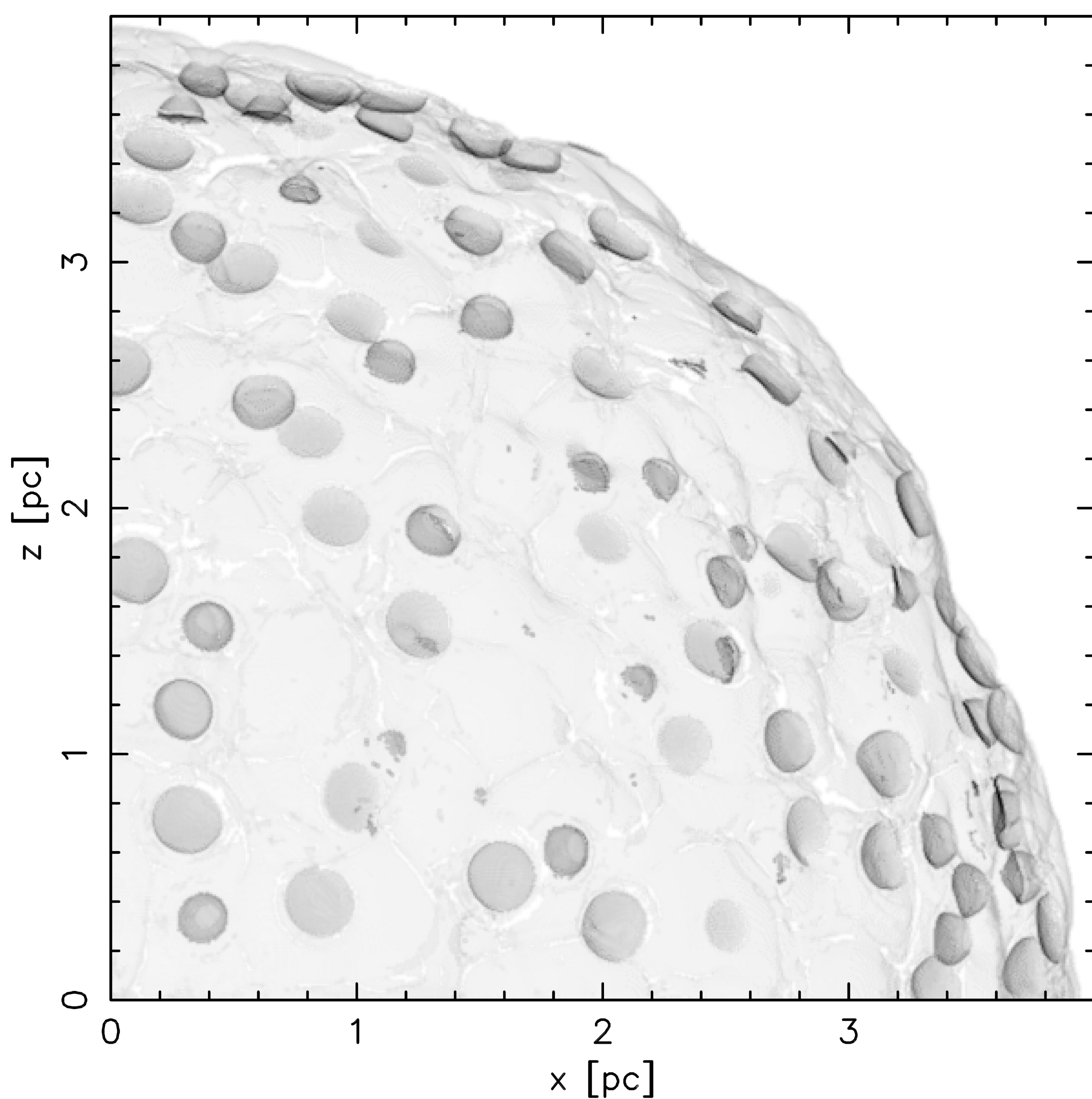
### QSA boosted by clumpy medium

At early stages of the simulation, the QSA electron synchrotron spectra look quite similar in the cases of uniform and clumpy ISM. However, shortly after the shock encounter the clumps, the QSA electron spectra gets boosted in energy by a factor of  $\sim 10^{2/3}$  which coincides with the magnetic field increase inside the clumps. Consequently, the steeper synchrotron spectrum extends up to  $\sim 5$  GHz and thus shifts the point where the DSA related part emerges from  $\sim 10$  to  $\sim 50$  GHz in the later stages. Such clumps are expected in the case of supernova Ib,c where a strong stellar wind precedes the explosion.

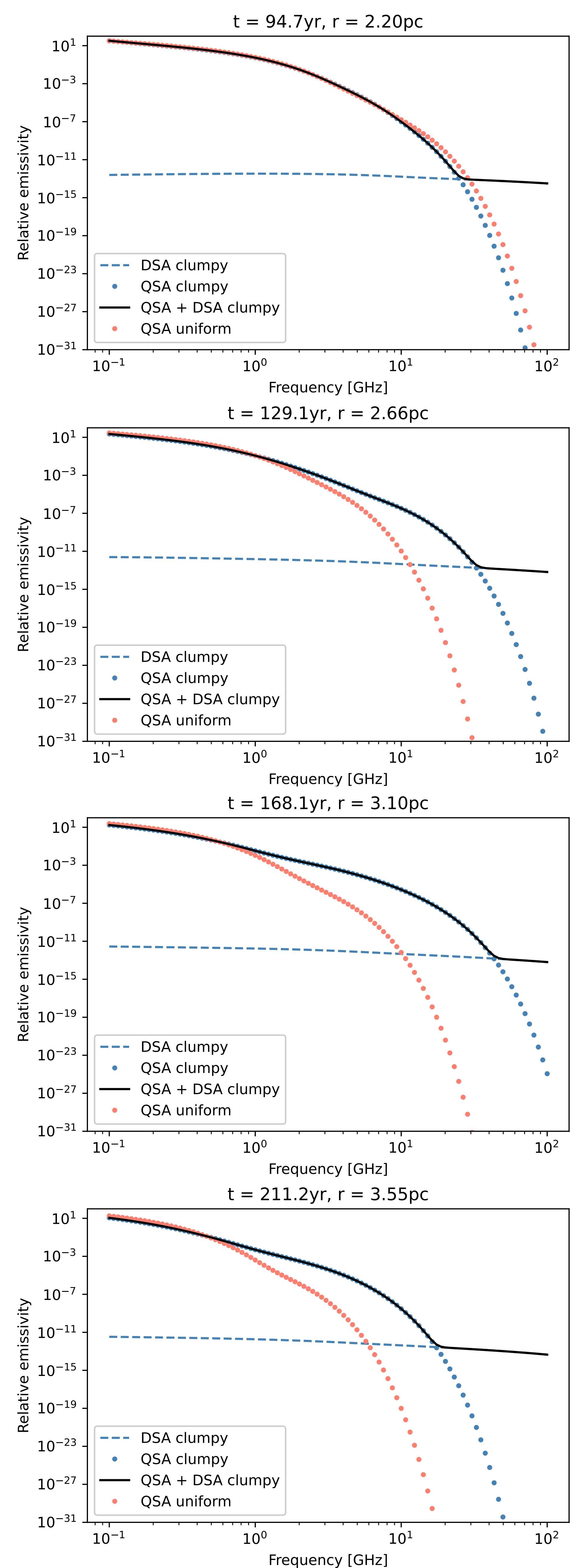
### Discussion

Steep synchrotron spectra are common to young SNRs (Bell et al. 2011). However, we expect the spectra to flatten at frequencies which correspond to the energy threshold for injection into DSA. At fast shocks with strongly amplified fields the injection energy for electrons is expected in GeV range (Zeković et al. 2024, Hemler et al. 2024). In our model we, however, did not include the DSA part which would flatten the spectra earlier than observed. Nevertheless, the aim of this study was to address the influence that clumps have on shock dynamics (by slowing down the shock), as well as, on the emission spectra due to the DSA regime. It turns out that although the Mach number drops when the shock enters the clumps, the assumed SLAMS stay superluminal because the shock is still fast during this early stage of the SNR evolution. The larger  $B_{clump}$  then boosts the electron acceleration by QSA to higher energies, which we observe as a proportional increase in the frequency where the spectrum falls off before flattening.

Our model thus have a potential application to more extreme shock environments with turbulent or clumpy medium, like those found at Novae, young SNRs, and jets of Active Galactic Nuclei.



**Figure 1** The density map of the SNR octant at the end of simulation. The shock is corrugated by the clumps on the scale of 0.3 pc. Variations on a larger scale are also visible.



**Figure 2** Electron synchrotron spectra at different stages of the SNR evolution. The uniform case is shown in red (QSA contribution), while the clumpy case is in blue where the dotted lines show QSA contribution and dashed lines show DSA contribution. The summed QSA+DSA spectra are represented by the black curves.

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