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Synchrotron polarization with a partially random magnetic field

General theory, and applications to IXPE observations of young supernova remnants

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MOTIVATIONS

- Polarization measurements in radio astronomy
  - dated since more than 60 yrs ago
  - complete information (all 4 Stokes parameters)

- In X-rays, since short ago, very little information
  (beyond integrated data on the Crab Nebula)

- "Imaging X-Ray Polarimetry Explorer" (IXPE - 2021)
  new window in X-ray astronomy
  (see Jacco Vink, and Niccolò Bucciantini, this meeting)

Observations of 3 young shell-type supernova remnants:
SN 1006, Tycho, Cas A

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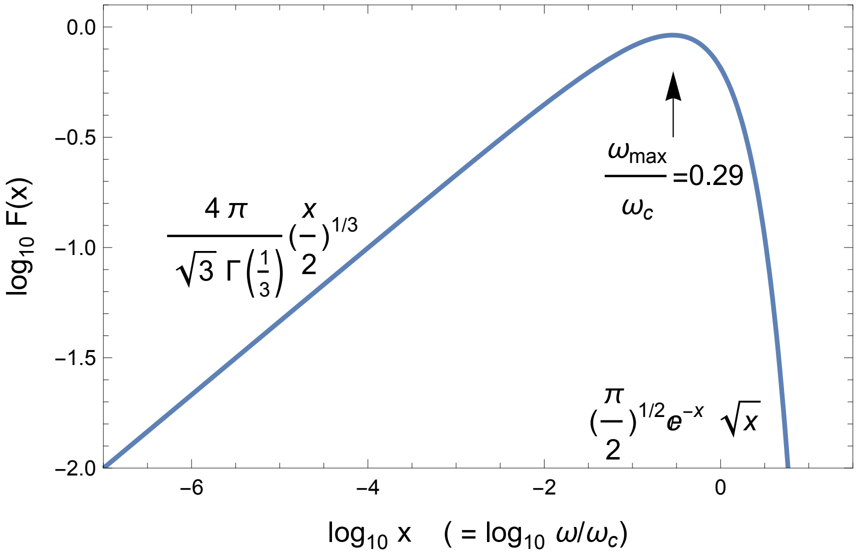
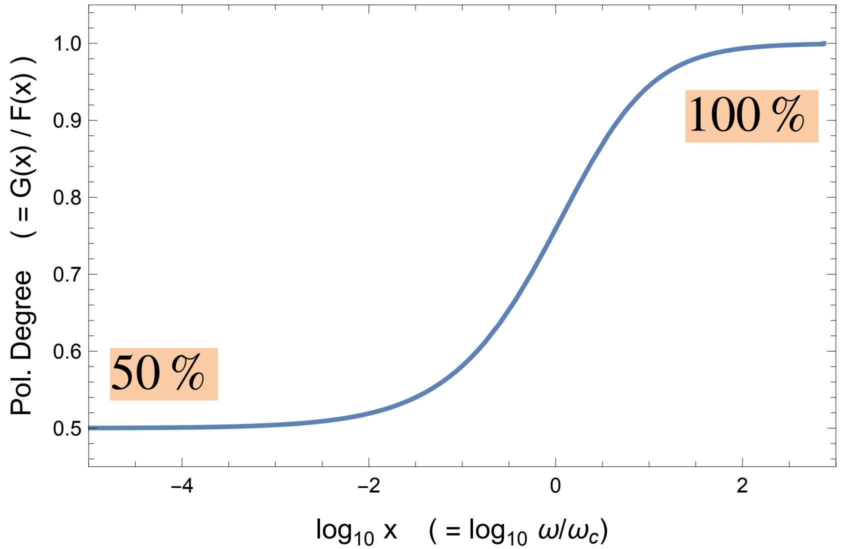
TEXT-BOOK THEORY OF SYNCHROTRON RADIATION MONO-ENERGETIC DISTRIBUTION

Broad-band spectrum



Highly polarised ($\perp \overrightarrow{B}_{\rm proj}$)



$$\omega_c = \frac{eB_{\rm proj}}{mc}\gamma^2 \,;\, x = \omega/\omega_c$$

$$\mathscr{I} = HB_{\rm proj}F(x) \text{ with: } F(x) = x\int_x^\infty K_{5/3}(z)\,dz$$

$$\mathscr{Q} = HB_{\rm proj}G(x) \text{ with: } G(x) = xK_{2/3}(x)$$

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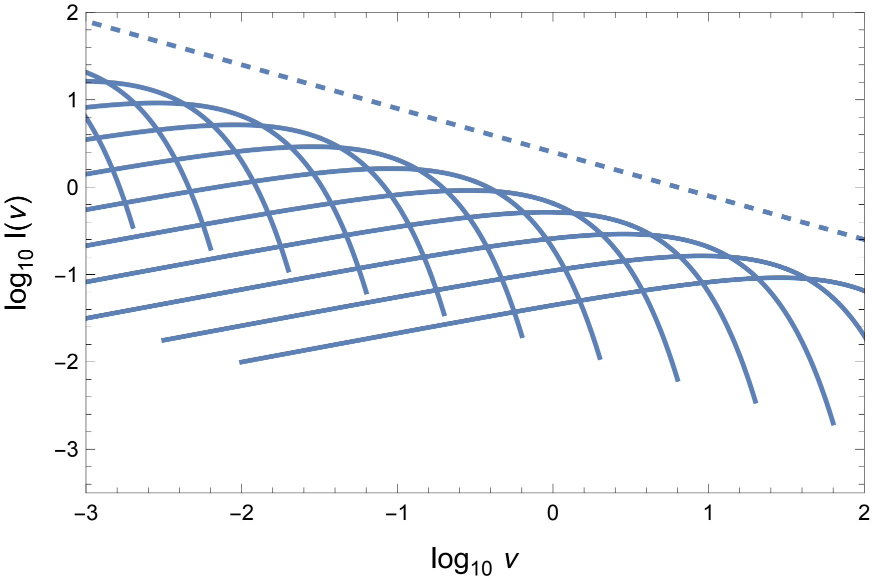
RADIATION FROM A DISTRIBUTION OF PARTICLES

Convolution of single particle emissions

Power-law energy distribution

$$n(\gamma) = A\gamma^{-s}$$

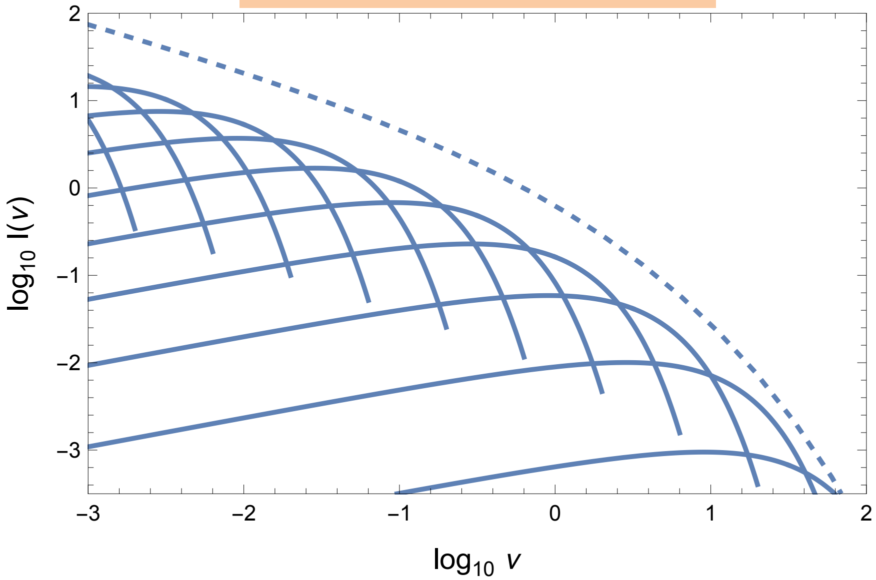
$$[\ \alpha = (s-1)/2\ ]$$



Power-law + cutoff energy distribution

$$n(\gamma) = A\gamma^{-s} \exp(-\left(\gamma/\gamma_{\rm cut}\right)^{\beta})$$

Parameters: s and β



Polarization Degree: $\Pi = \dfrac{s+1}{s+7/3}$. Higher PD when steeper slope.

THE PROBLEM

- Classical theory valid for homogenous fields

- BUT homogeneous magnetic fields are VERY RARE

- Superposition of differently oriented magnetic fields:
    - either due to large scale variations along the line of sight
    - or due to limited spatial resolution
    - or in the presence of “really” random fields

How to make the most appropriate use of the polarization information
- Radio observations
- X-ray observations

SOLUTION IN THE RADIO SPECTRAL RANGE
(= pure power-law) (Bandiera & Petruk 2016)

- Homogeneous field ($\bar{B} \parallel \hat{y}$) + isotropic random field:

$$\mathscr{P}_x(B_x) = \frac{1}{\sqrt{2\,\pi}\,\sigma} \exp\left(-\frac{B_x^2}{2\sigma^2}\right); \quad \mathscr{P}_y(B_y) = \frac{1}{\sqrt{2\,\pi}\,\sigma} \exp\left(-\frac{(B_y - \bar{B})^2}{2\sigma^2}\right)$$

- Anisotropic random field:

$$\mathscr{P}_x(B_x) = \frac{1}{\sqrt{2\,\pi}\,\sigma_x} \exp\left(-\frac{B_x^2}{2\sigma_x^2}\right); \quad \mathscr{P}_y(B_y) = \frac{1}{\sqrt{2\,\pi}\,\sigma_y} \exp\left(-\frac{B_y^2}{2\sigma_y^2}\right)$$

ASSUMPTION: Gaussian distributions

CALCULATION OF THE AVERAGED STOKES PARAMETERS

(from now on $B_{\mathrm{proj}} \rightarrow B$)

Single particle:
$\mathcal{I} = H\,B\,F(x)$
$\mathcal{Q} = H\,B\,G(x)$

$$\langle \mathcal{I} \rangle = H \iiint B \qquad F\left(\frac{mc}{eB}\frac{\omega}{\gamma^2}\right) n(\gamma) \mathcal{P}_{x,y}(B_x, B_y)\, d\gamma\, dB_x\, dB_y$$

$$\langle \mathcal{Q} \rangle = H \iiint B \frac{B_y^2 - B_x^2}{B_y^2 + B_x^2} G\left(\frac{mc}{eB}\frac{\omega}{\gamma^2}\right) n(\gamma) \mathcal{P}_{x,y}(B_x, B_y)\, d\gamma\, dB_x\, dB_y$$

Where $\cos(2\chi) = \frac{B_y^2 - B_x^2}{B_y^2 + B_x^2}$ is needed to orient correctly the individual $\mathcal{Q}$ values

- Homogeneous field + isotropic random field:

$$\mathcal{I}(\omega) = \frac{s+7/3}{s+1} H_0 \omega^{-(s-1)/2} \bar{B}^{(1+s)/2} \left\{ \Gamma\left(\frac{5+s}{4}\right)\left(\frac{\bar{B}}{\sqrt{2}\sigma}\right)^{-(1+s)/2} {}_1F_1\left(-\frac{1+s}{4}, 1, -\frac{\bar{B}^2}{2\sigma^2}\right)\right\}$$

$$\mathcal{Q}(\omega) = H_0 \omega^{-(s-1)/2} \bar{B}^{(1+s)/2} \left\{ \frac{1}{2}\Gamma\left(\frac{9+s}{4}\right)\left(\frac{\bar{B}}{\sqrt{2}\sigma}\right)^{-(1+s)/2} {}_1F_1\left(\frac{3-s}{4}, 3, -\frac{\bar{B}^2}{2\sigma^2}\right)\right\}$$

- Anisotropic random field ($\sigma_{\rm eff}^2 = \frac{2\sigma_x^2\sigma_y^2}{\sigma_x^2+\sigma_y^2}$; $f_{\rm anis} = \frac{\sigma_y^2-\sigma_x^2}{\sigma_x^2+\sigma_y^2}$): $-1 < f_{\rm anis} < +1$

$$\mathcal{I}(\omega) = \frac{s+7/3}{s+1} H_0 \omega^{-(s-1)/2} (2\sigma_{\rm eff}^2)^{(1+s)/4} \left\{ \sqrt{1-f_{\rm anis}^2}\,\Gamma\left(\frac{5+s}{4}\right) {}_2F_1\left(\frac{5+s}{4}, \frac{9+s}{4}, 1, f_{\rm anis}^2\right)\right\}$$

$$\mathcal{Q}(\omega) = H_0 \omega^{-(s-1)/2} (2\sigma_{\rm eff}^2)^{(1+s)/4} \left\{ \frac{f_{\rm anis}}{2}\sqrt{1-f_{\rm anis}^2}\,\Gamma\left(\frac{9+s}{4}\right) {}_2F_1\left(\frac{9+s}{8}, \frac{13+s}{8}, 2, f_{\rm anis}^2\right)\right\}$$

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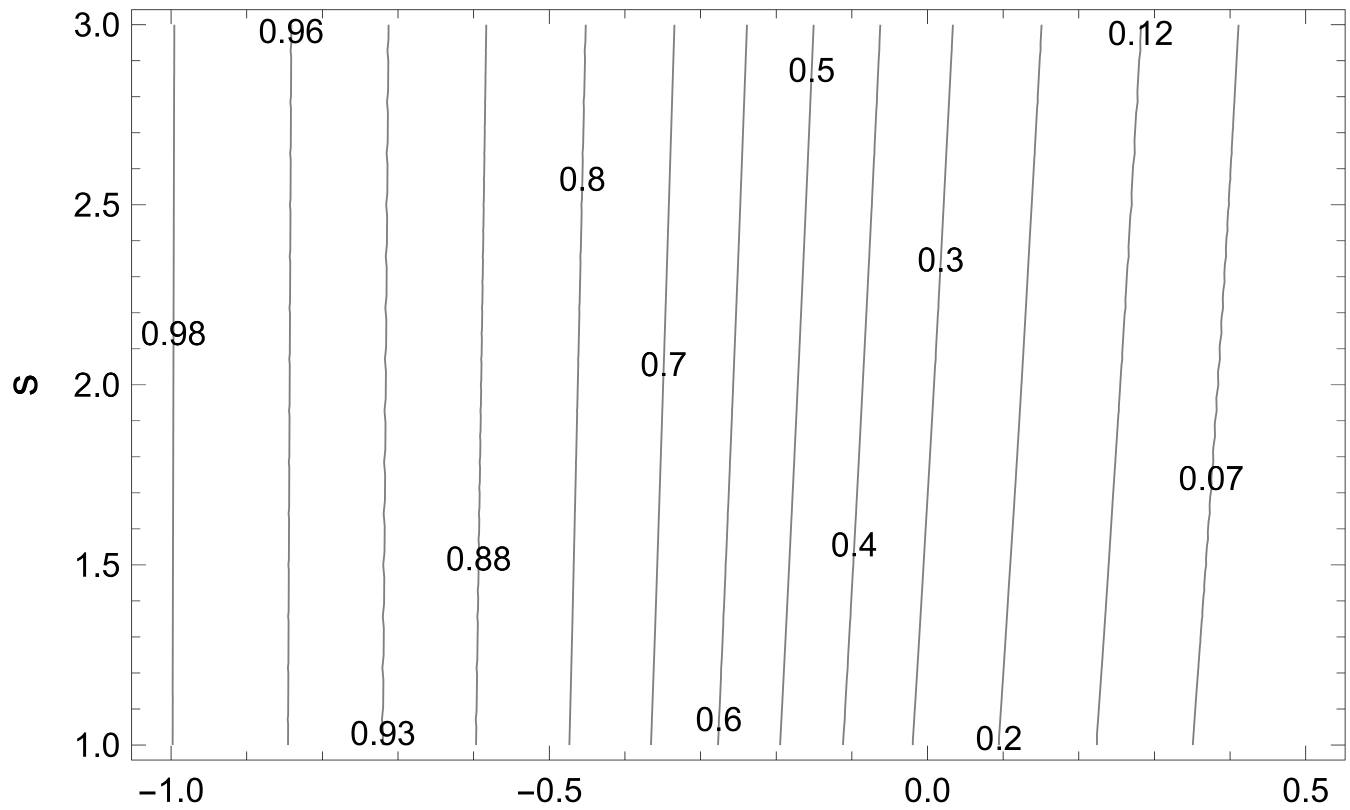
Homogeneous + isotropic random

(from Bandiera & Petruk 2016)

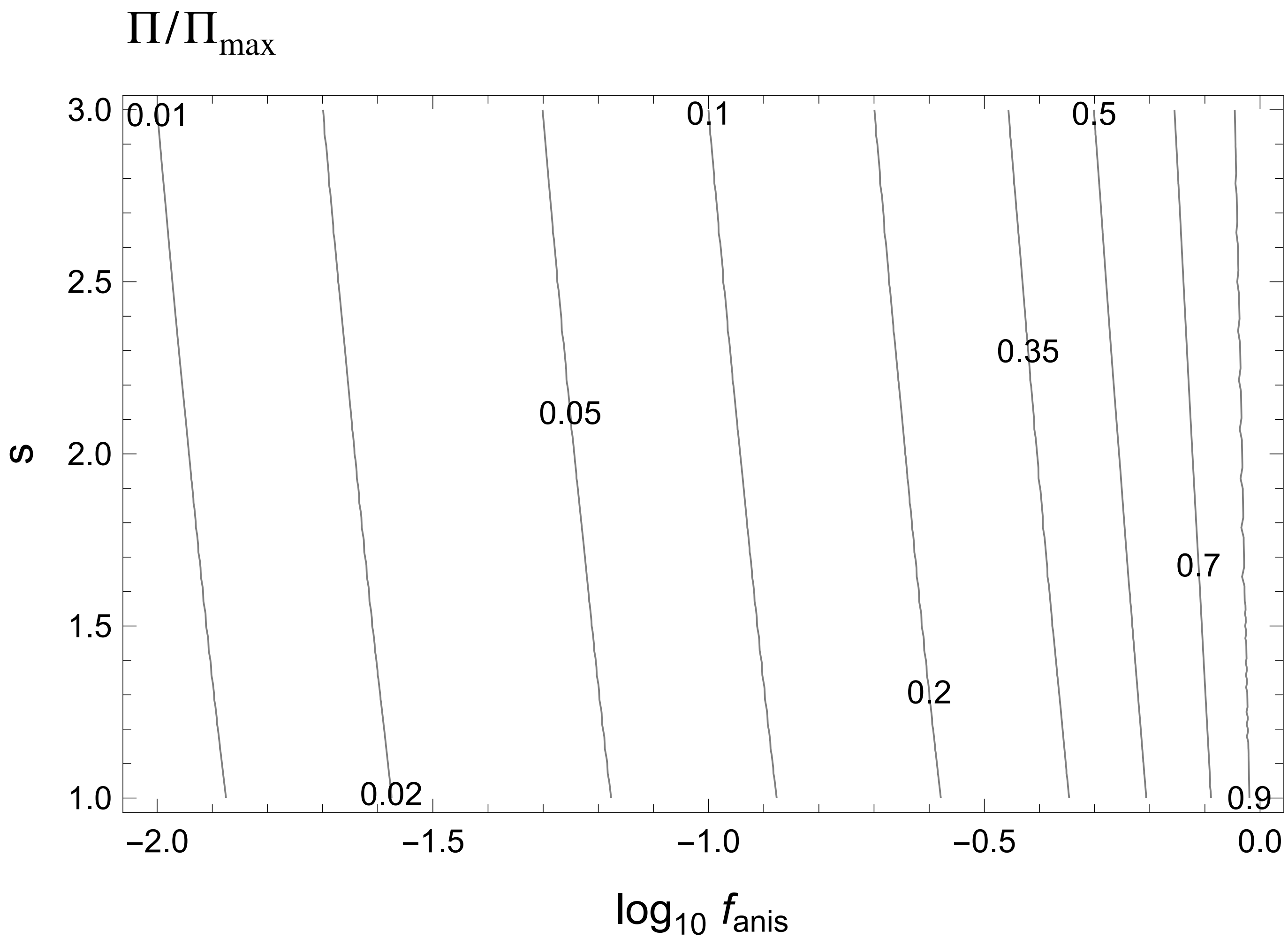
$\Pi/\Pi_{\max}$



$\log_{10} \sigma/\overline{B}$

$$\Pi_{\max} = \frac{s+1}{s+7/3}$$

Anisotropic random



$$f_{\mathrm{anis}} = \frac{\sigma_y^2 - \sigma_x^2}{\sigma_x^2 + \sigma_y^2}$$

BEYOND THE POWER LAW PROFILE

(Bandiera & Petruk 2024, accepted on A&A, arXiv:2405.14534)

Generalized energy distribution: $n(\gamma) = A\gamma^{-s}\exp(-(\gamma/\gamma_{\rm cut})^{\beta})$

$$\nu_{\rm br} = \frac{0.29}{2\pi}\omega_{\rm cut} = \frac{0.29}{2\pi}\frac{eB_{\rm proj}}{mc}\gamma_{\rm cut}^2$$

$s \simeq 2$ for a strong shock

NO ANALYTIC SOLUTION

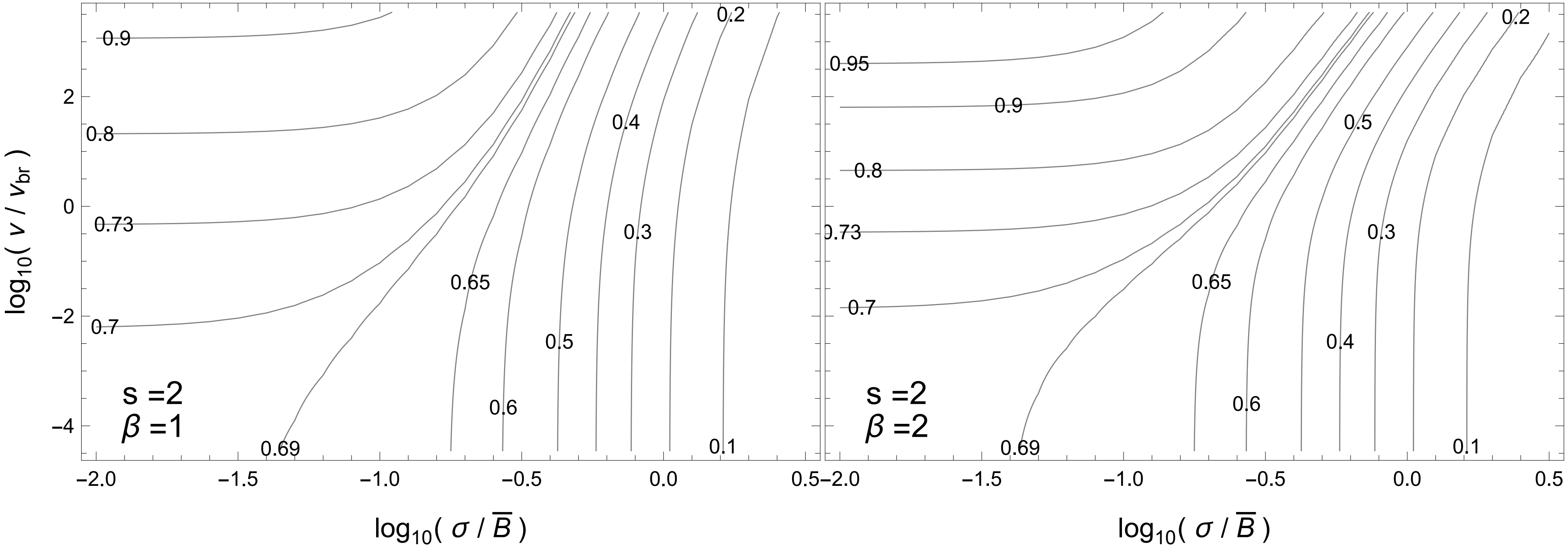
Fully numerical solution: multiple integrations for each model;
unsuited for a thorough investigation in the parameter space.

Much faster calculations, with a two-phase method:

1. Integration on the particle energy distribution.
   For any choice of s and β. Independent of the magnetic field distribution.
2. Integration on the magnetic field distribution.

Dependence of the polarization degree:

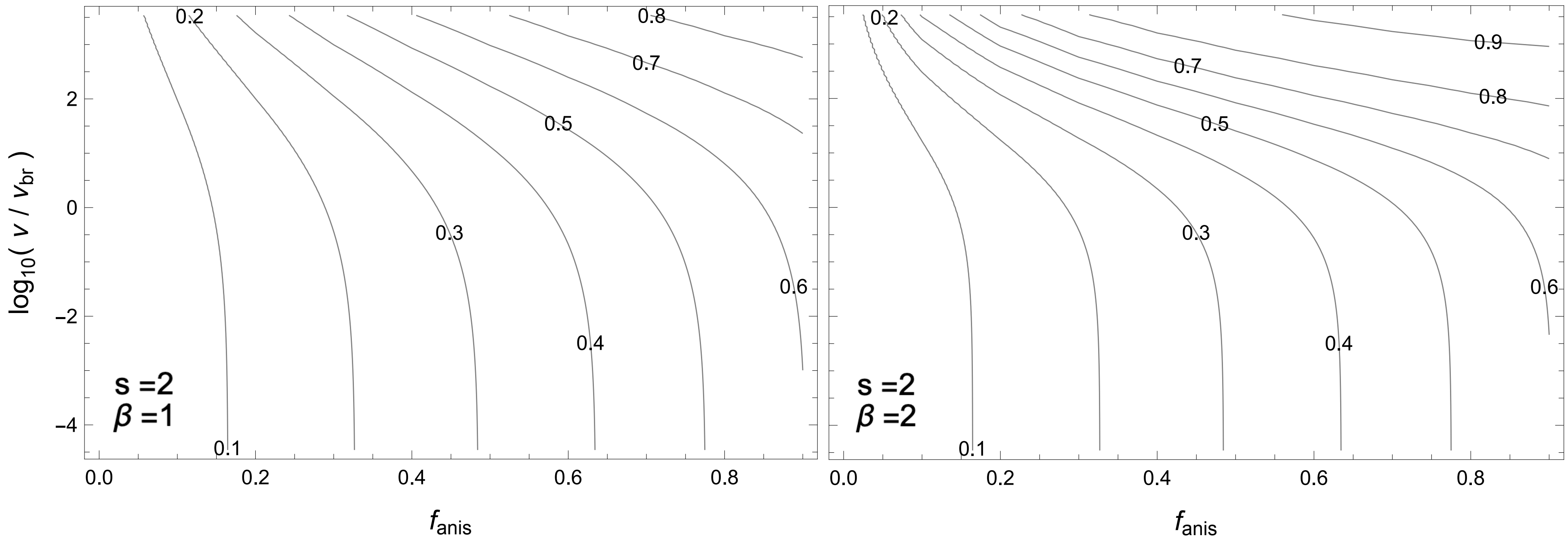
1. on the magnitude of the random field
2. on the position in the spectrum



$$\Pi_{max}(s = 2) = 0.692$$

Same, for an anisotropic random field.
Dependence of the polarization degree:

1. on the level of anisotropy of the random field
2. on the location in the spectrum



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WITH A MORE OBSERVATIONAL FLAVOUR

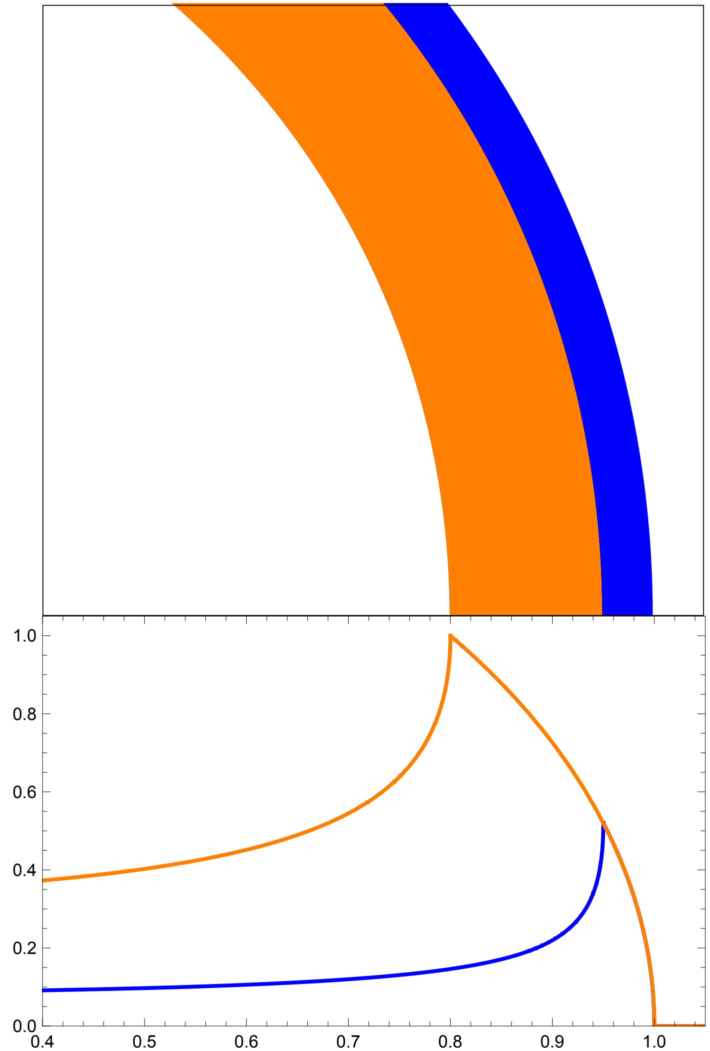
The measured $\nu_{\rm break}$ depends:

- on the model assumed (i.e. assumed value for β)

- on the spectral range used

- on the assumption that "the emission in different spectral ranges comes from the SAME regions"

$\nu_{\rm break}$ is underestimated $\Leftarrow$



CAN WE COMBINE RADIO AND X-RAY DATA FOR A GLOBAL SPECTRAL FIT ?

Synchrotron emitting filaments. Very thin, in the X-rays.

Either:
- Magnetic field disappears through some damping mechanism

(Pohl, Yan & Lazarian 2005)

BUT WHY different behaviour between radio and X-rays ?

Or:
- X-ray emitting electrons disappear because of radiative losses

(Link & Laming 2003)

BUT the downstream magnetic field must be high

($B \simeq 50 \div 200\,\mu$G for the historical SNRs)

WHAT CAN WE MEASURE

More direct observational quantities, in a given spectral band, are:

- the polarization fraction (in radio as well in the X-rays)

- the "local" spectral index (local, i.e. in that specific specific band)

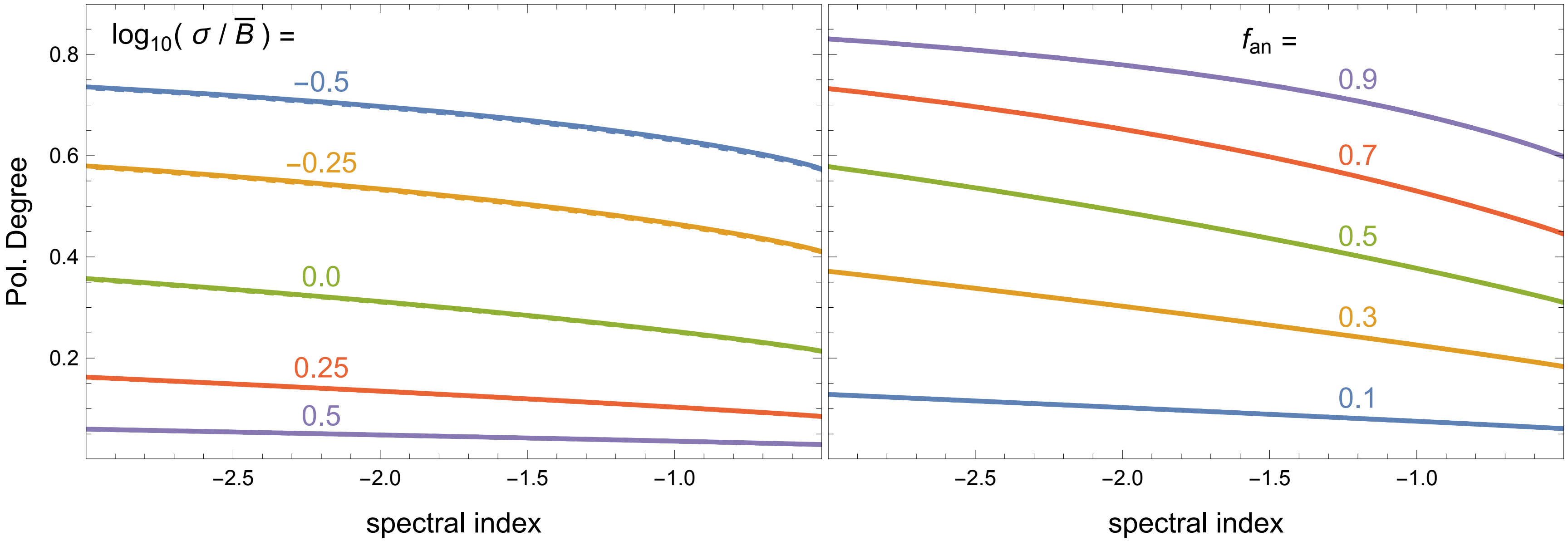
and, beyond them:

- the direction of polarization

- the "curvature" of the spectrum

Polarization degree vs local spectral index

$(s = 2)$ homogeneous+isotropic anisotropic

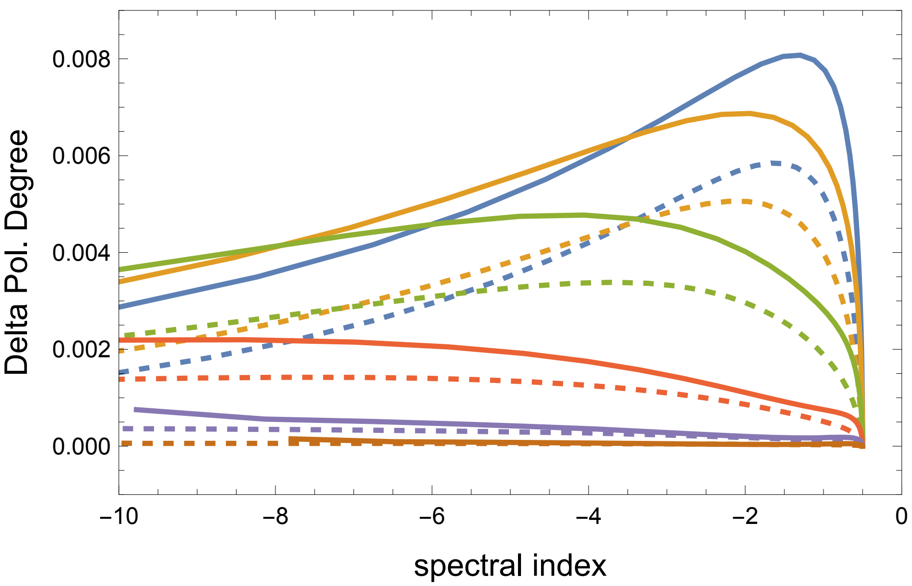


[dashed: $\beta = 1$; solid: $\beta = 2$]

Almost independent of the value of β !

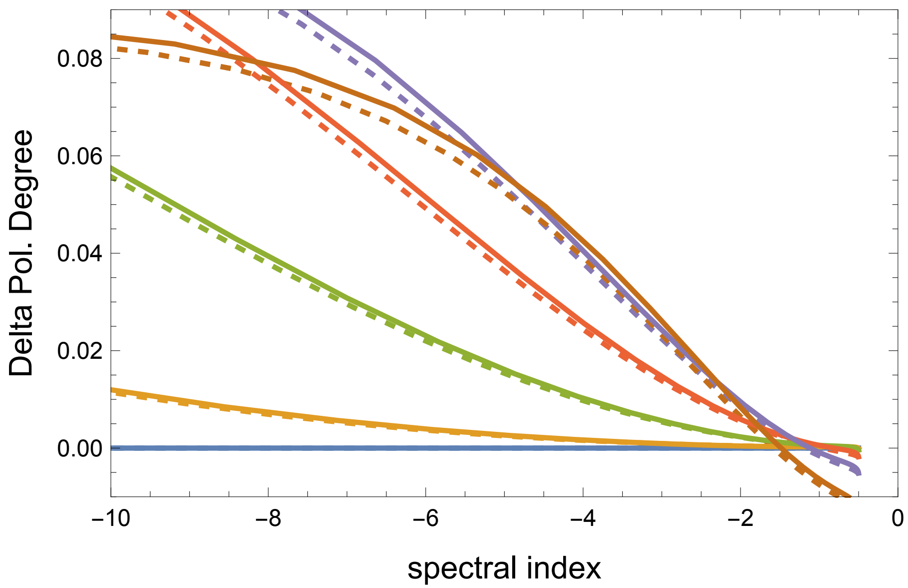
CAN WE STILL USE OUR ANALYTICAL FORMULAS?

homogeneous+isotropic



$\log_{10}(\sigma/\bar{B})$ = -0.5,-0.25,0.0,0.25, 0.5

anisotropic



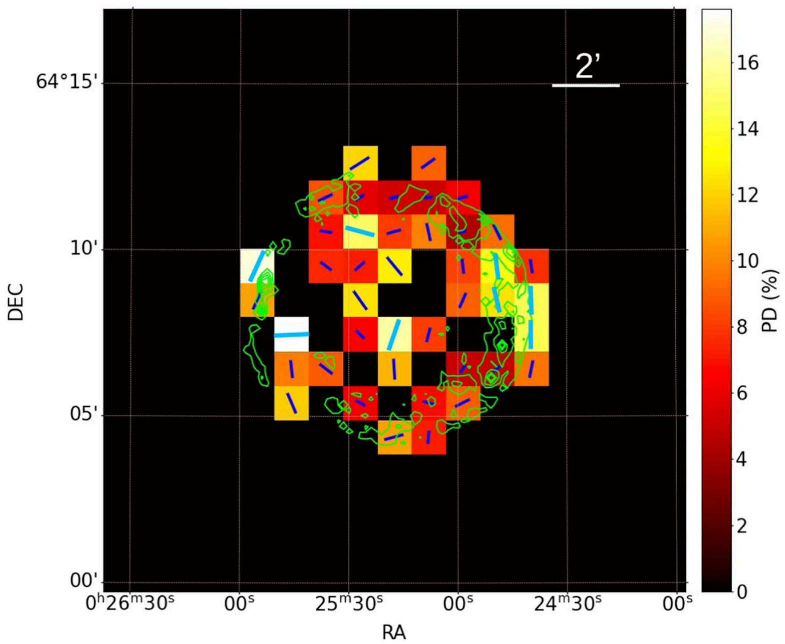
$f_{\rm anis}$ = 0.1, 0.3, 0.5, 0.7, 0.9

[$s = 2$] [dashed: $\beta = 1$; solid: $\beta = 2$]

Analytic formulae - good approximations also in cases with break

SHELL-TYPE SUPERNOVA REMNANTS AS SEEN FROM IXPE

(Tycho, SN1006, Cas A - see Jacco Vink, this meeting)

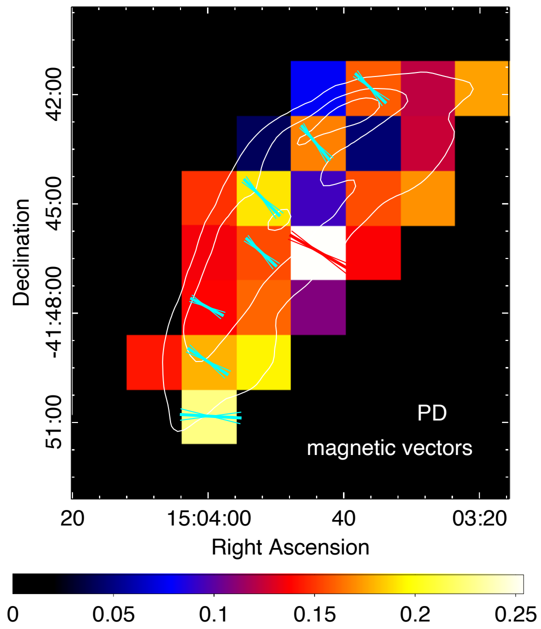


Tycho (Ferrazzoli+ 2023)
[(electric) polarization]

⇐

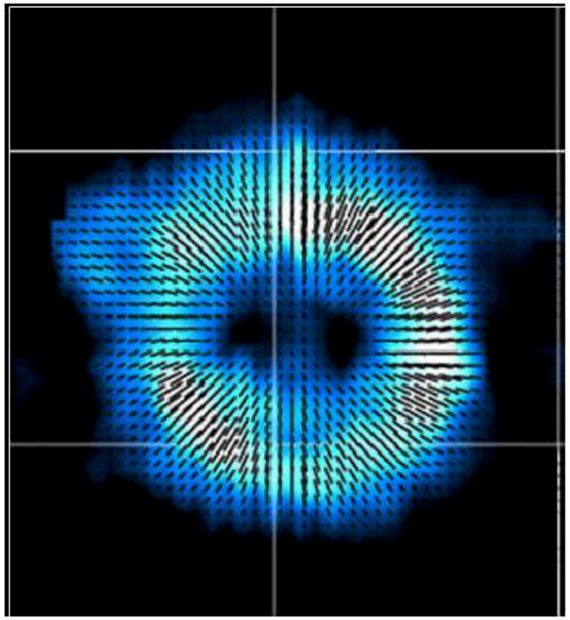
SN1006 (Zhou+ 2023)
[magnetic vectors]

⇒

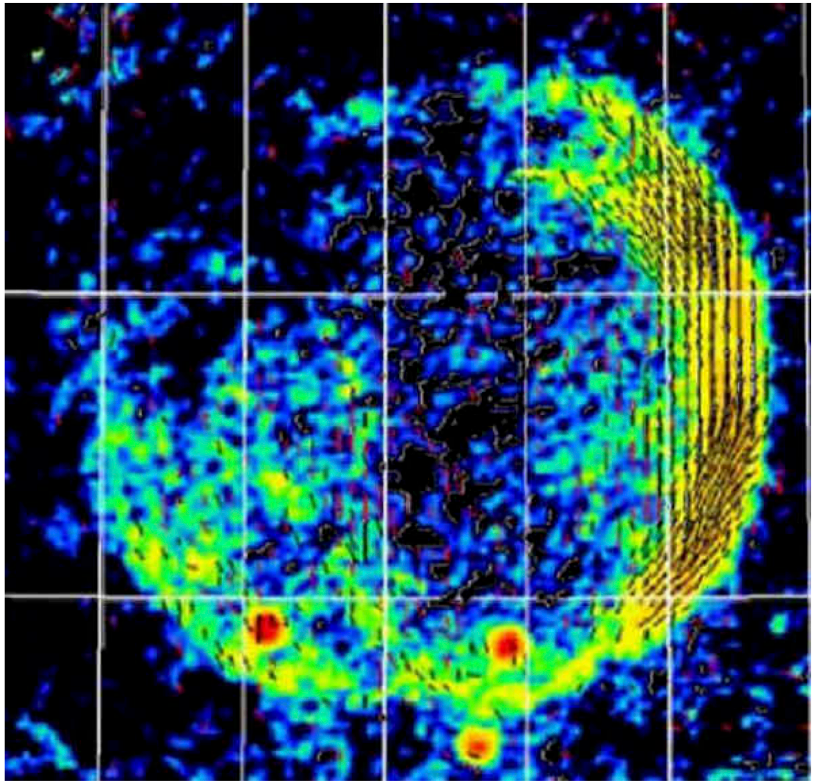


Preferentially radial orientation of the magnetic fields

Radially-oriented magnetic fields in young supernova remnants
Tangentially-oriented field in older supernova remnants



Cas A



CTB1

(W. Reich - 100 m Effelsberg Radio Telescope)

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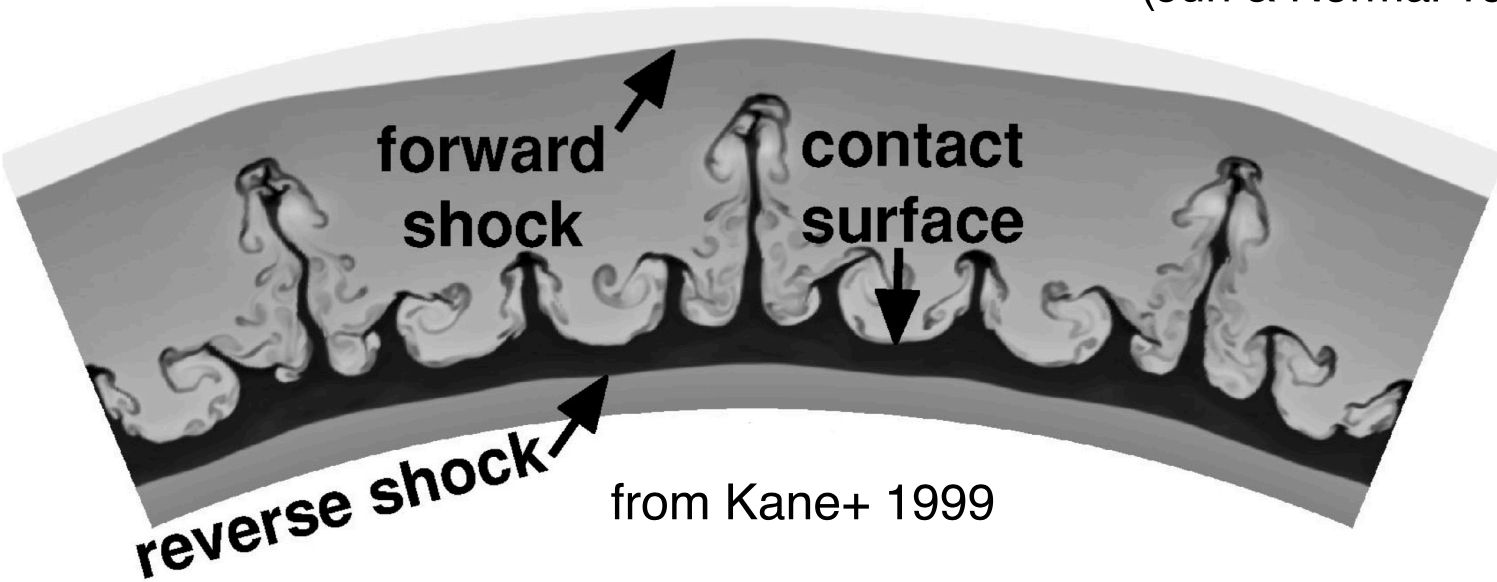
Compression at the supernova forward shock
⇒ Tangentially oriented magnetic fields

Radially oriented magnetic fields ⇒ STRETCHED ???

Effect of some hydrodynamic instability ?
Rayleigh-Taylor / Richtmyer-Meshkov instabilities
(Jun & Normal 1996; Inoue et al 2013)



from Kane+ 1999

TYCHO'S SUPERNOVA REMNANT

(Ferrazzoli+ 2023)

Table 1
Results in the 3–6 keV Energy Band for Each Region of Interest

Region	Q/I (%)	U/I (%)	σ	CL (%)	PD (%)	PD_{Corr} (%)	P.A. (°)
All	3.5 ± 0.7	0.1 ± 0.7	5.0	>99.99	3.5 ± 0.7	9.1 ± 2.0	$1 \pm 6(7)$
Rim (g)	4.8 ± 0.8	-0.4 ± 0.8	6.0	>99.99	4.8 ± 0.8	11.9 ± 2.2	$-2 \pm 5(5)$
West, χ_2^2 (f)	9.7 ± 1.8	-2.6 ± 1.8	5.6	>99.99	10.0 ± 1.8	23.4 ± 4.2	$-7 \pm 5(5)$
West stripes (b)	7.1 ± 2.0	-1.7 ± 2.0	3.7	99.87	7.3 ± 2.0	13.9 ± 3.8	$-7 \pm 8(8)$
East (e)	7.3 ± 3.2	-3.5 ± 3.2	2.5	95.99	<16	<32	NC
Northeast (a)	5.0 ± 2.3	2.0 ± 2.3	2.4	94.07	<11	<36	NC
South stripes (c)	2.9 ± 3.6	0.0 ± 3.6	0.8	27.64	<12	<29	NC
Arch (d)	3.2 ± 4.3	0.5 ± 4.3	0.8	24.47	<14	<35	NC

Using B&P2016 formulae for ordered B + isotropic δB $(\delta B = \sqrt{3}\,\sigma)$

Rim: PD $11.9\,\%$ + photon index 2.82 ± 0.02 $\Rightarrow$ $\delta B/B = 3.4 \pm 0.3$

West: PD $23.4\,\%$ + photon index 2.90 ± 0.04 $\Rightarrow$ $\delta B/B = 2.2 \pm 0.4$

While for Tycho must be $\delta B/B \sim 20$ downstream (Morlino & Caprioli 2022)

WHY SUCH A LOW δB ?

Use of the analytic formulae for a power-law spectrum
(OK, within $\sim 1\,\%$)

BUT

Likely effect of anisotropic random B
(and dominant over the homogeneous B)

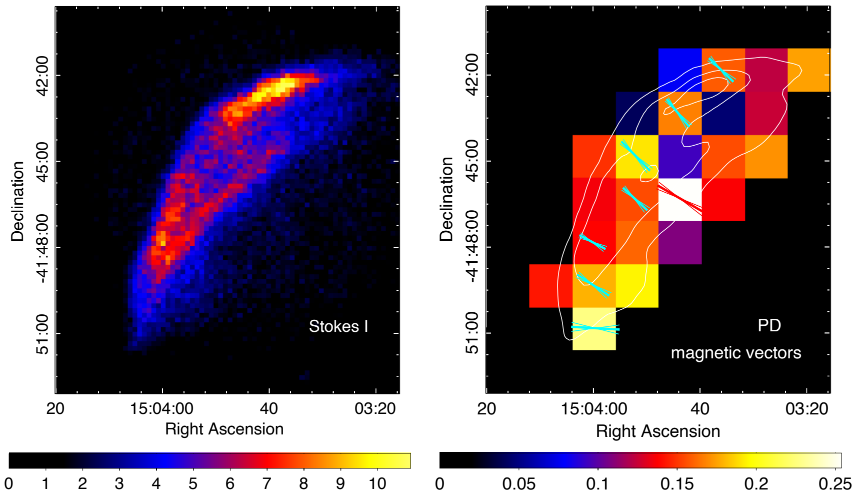
From our formulae:

Rim: PD $11.9\,\%$ + photon index 2.82 ± 0.02 $\Rightarrow$ $\sigma_r/\sigma_t = 1.13 \pm 0.03$

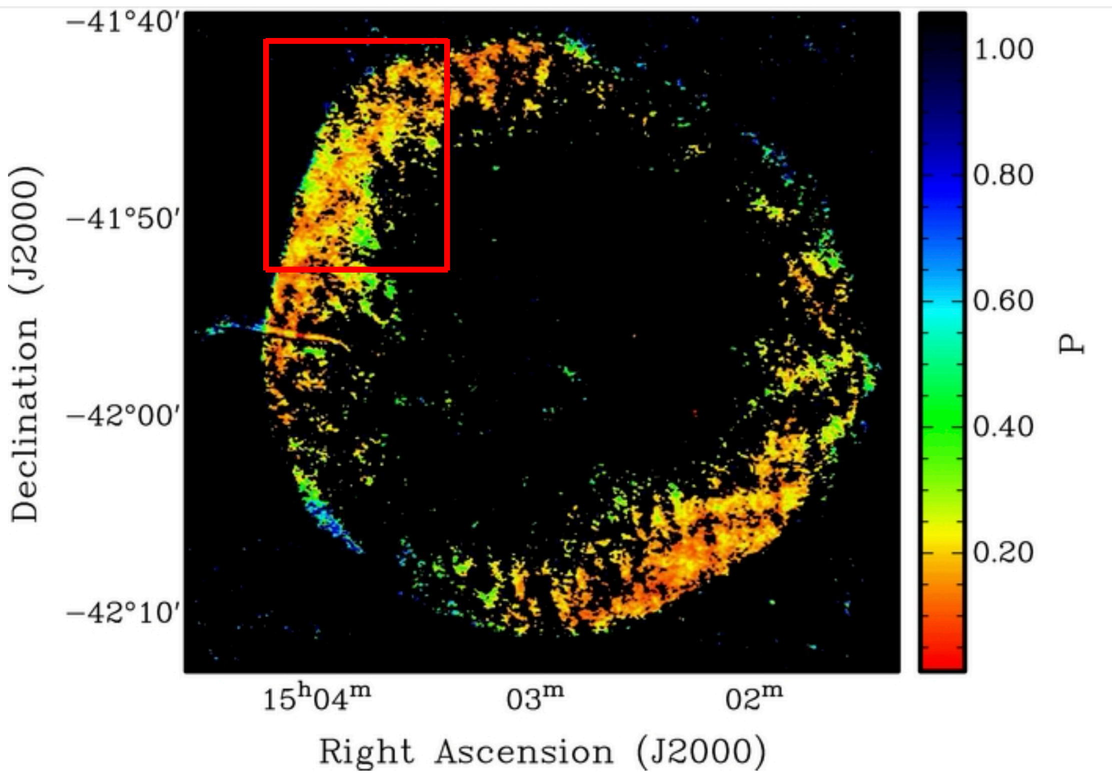
West: PD $23.4\,\%$ + photon index 2.90 ± 0.04 $\Rightarrow$ $\sigma_r/\sigma_t = 1.28 \pm 0.06$

SN1006 SUPERNOVA REMNANT

IXPE (Zhou et al. 2023)



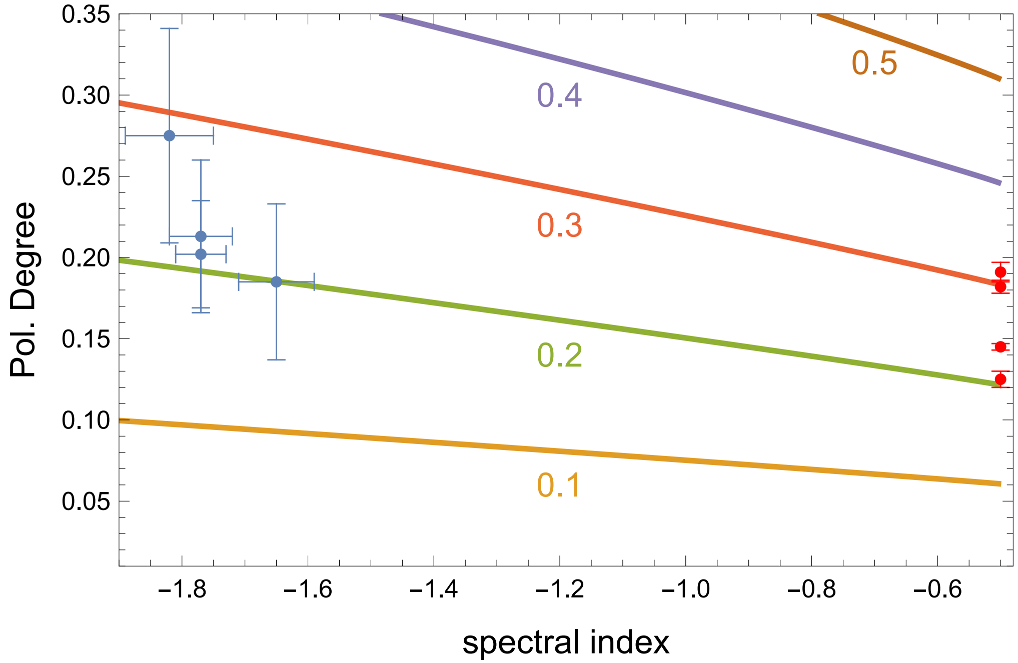
Radio (Reynoso et al. 2013)



Using the X-ray polarization information for SN 1006

Table 1. Polarization results of 5 regions by IXPE and radio observations. (Zhou et al 2023)

Region	IXPE (polarimetric)			IXPE (spectropolarimetric)			radio	
	PD (%)	PA (°)	σ	PD (%)	PA (°)	Γ	PD_r (%)	PA_r (°)
Shell	22.4 ± 3.5	-45.4 ± 4.5	6.3	20.2 ± 3.3	-49.4 ± 4.7	2.77 ± 0.04	14.5 ± 0.2	-36.3 ± 0.4
A	20.7 ± 4.5	-49.5 ± 6.2	4.6	21.3 ± 4.7	-56.1 ± 6.2	2.77 ± 0.05	12.5 ± 0.5	-58.4 ± 1.0
B	22.8 ± 4.9	-43.7 ± 6.2	4.6	18.5 ± 4.8	-44.8 ± 7.5	2.65 ± 0.06	19.1 ± 0.6	-28.2 ± 0.9
C	21.9 ± 6.4	-40.0 ± 8.3	3.4	27.5 ± 6.6	-45.3 ± 6.5	2.82 ± 0.07	18.2 ± 0.4	-28.0 ± 0.7
~~D~~	~~< 38.8~~	~~undetermined~~	~~2.5~~	~~< 31.8~~	~~undetermined~~	~~3.11 ± 0.09~~	~~13.9 ± 0.4~~	~~-39.8 ± 0.8~~



Anisotropic random field

Consistent with

$$f_{\rm anis} = 0.21 \div 0.29$$

$$(\sigma_r = 1.22 \div 1.35\, \sigma_t)$$

Constraint on HD models.

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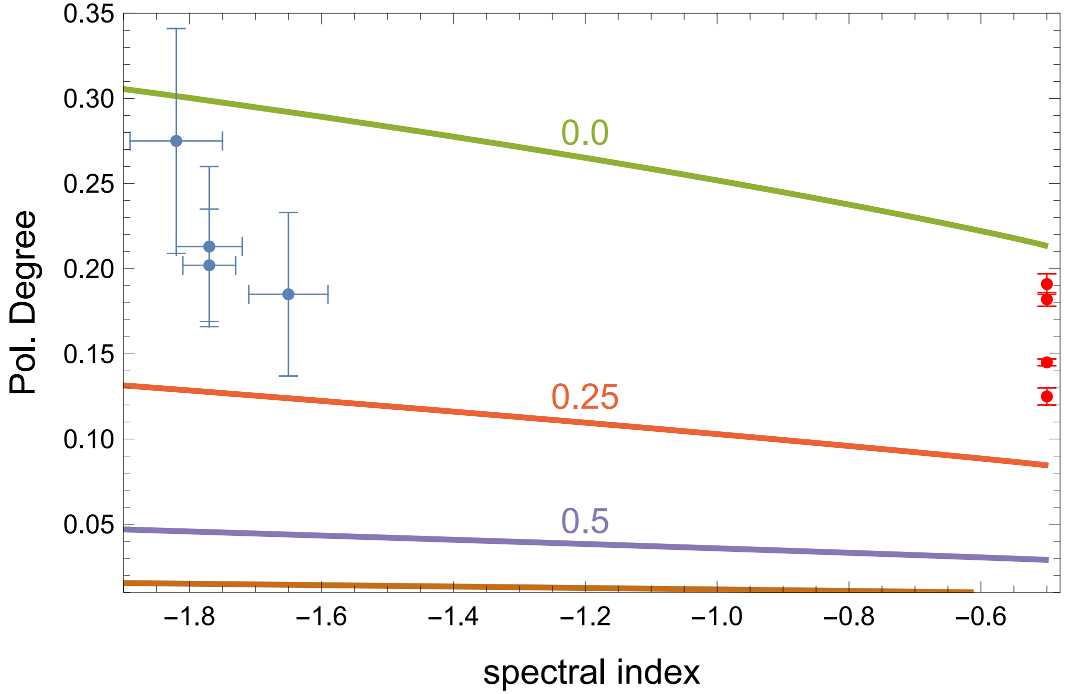
Homogeneous field + isotropic random field models ?



$\log_{10} \sigma/\bar{B} = 0.03 \div 0.14$

(i.e. $\sigma = 1.07 \div 1.38\,\bar{B}$)

(i.e. $\delta B = 1.85 \div 2.39\,\bar{B}$)

Moderate field amplification ?

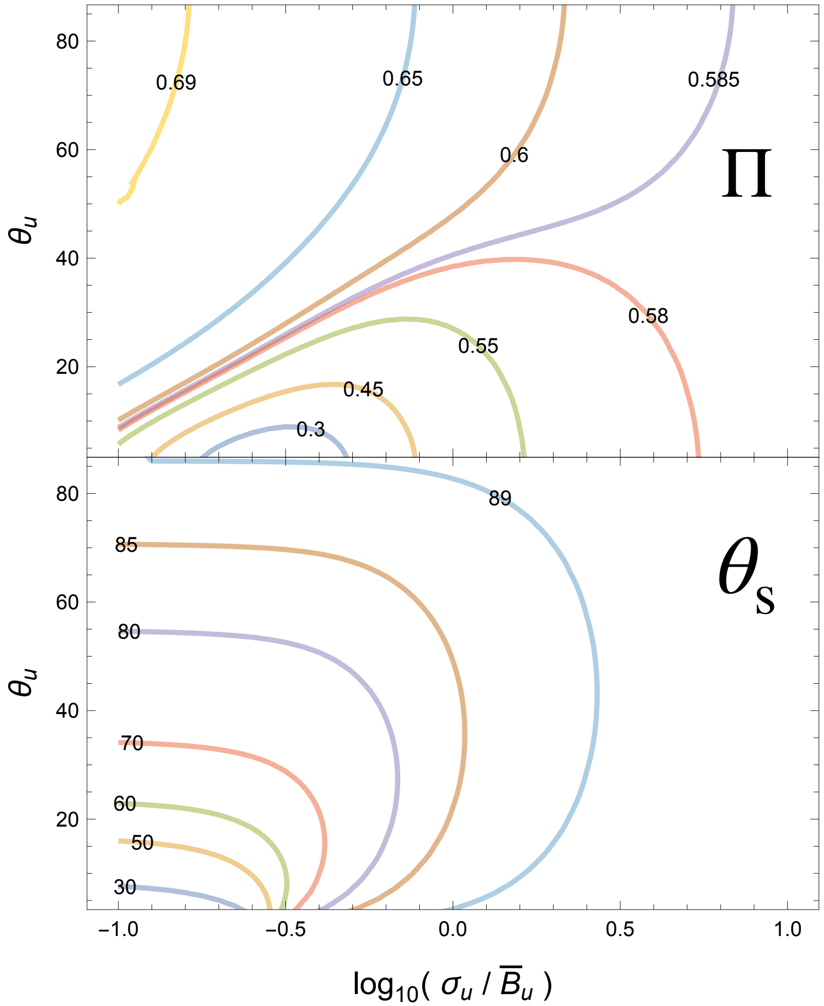
Inconsistent with standard scenario

A MODEL FOR OLDER SUPERNOVA REMNANTS IN A TURBULENT AMBIENT MEDIUM

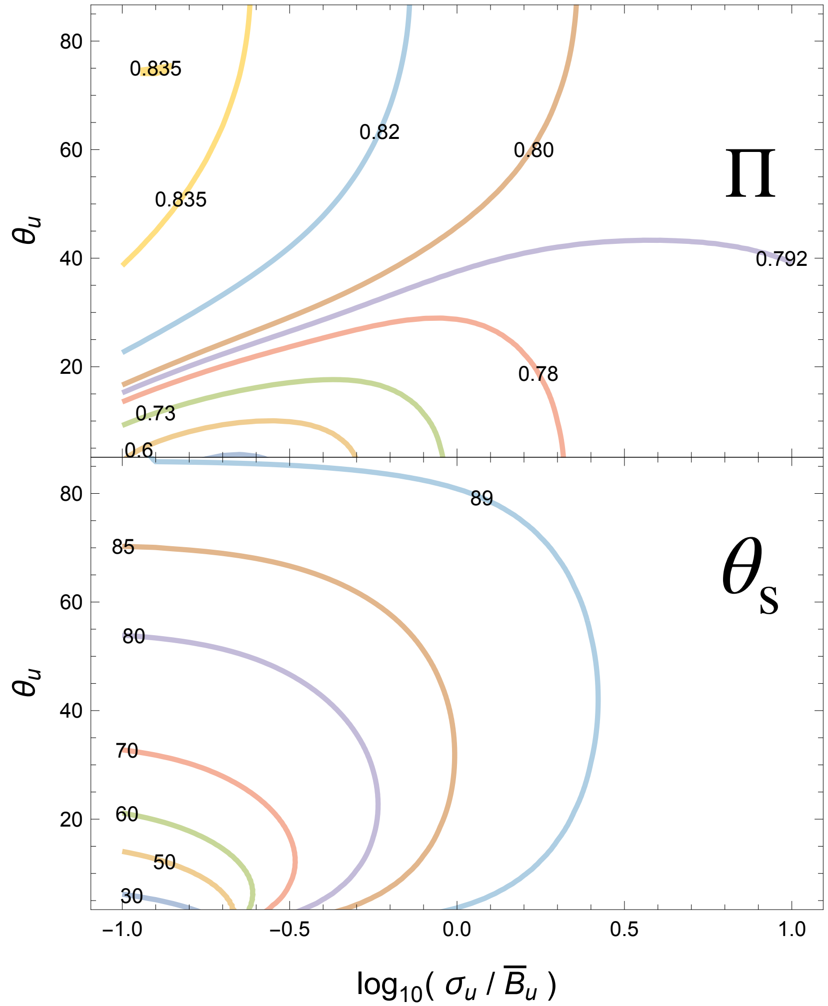
Homogeneous field + anisotropic random field

Strong shock ($\kappa = 4$)
homogenous MF ($\bar{B}_{\mathrm{u}}$) + isotropic random MF (σ_{u})
in the upstream
and PURE MAGNETIC FIELD COMPRESSION

Different choices for the direction (θ_{u}) of $\bar{B}_{\mathrm{u}}$.



$s = 2$ (i.e. $\alpha = -0.5$)



$s = 6$ (i.e. $\alpha = -2.5$)

RANDOM B FIELD $\neq$ TURBULENT B FIELD

Apart from correlations (in intensity and/or polarization) between different locations (see e.g. Shimoda+ 2018, for Tycho SNR),

even along a single line of sight, not at all obvious if it can be approximated with a Gaussian distribution function.

Integrating over a PDF = assuming an infinite number of cells
BUT
in turbulence most of the power at larger scales
"effective" number of cells is not very large

Simulation valid for the radio range (power law, with $s = 2$)

Kolmogorov power spectrum, for $k > k_1$ (in units of 1 / LOS)

WORK IN PROGRESS / PRELIMINARY

Fast but simplified method,
still to be validated by comparison
with fully (3-D) simulations

(see Poster 5.4, by Luca Del Zanna)

Integrated effect along the line of sight.

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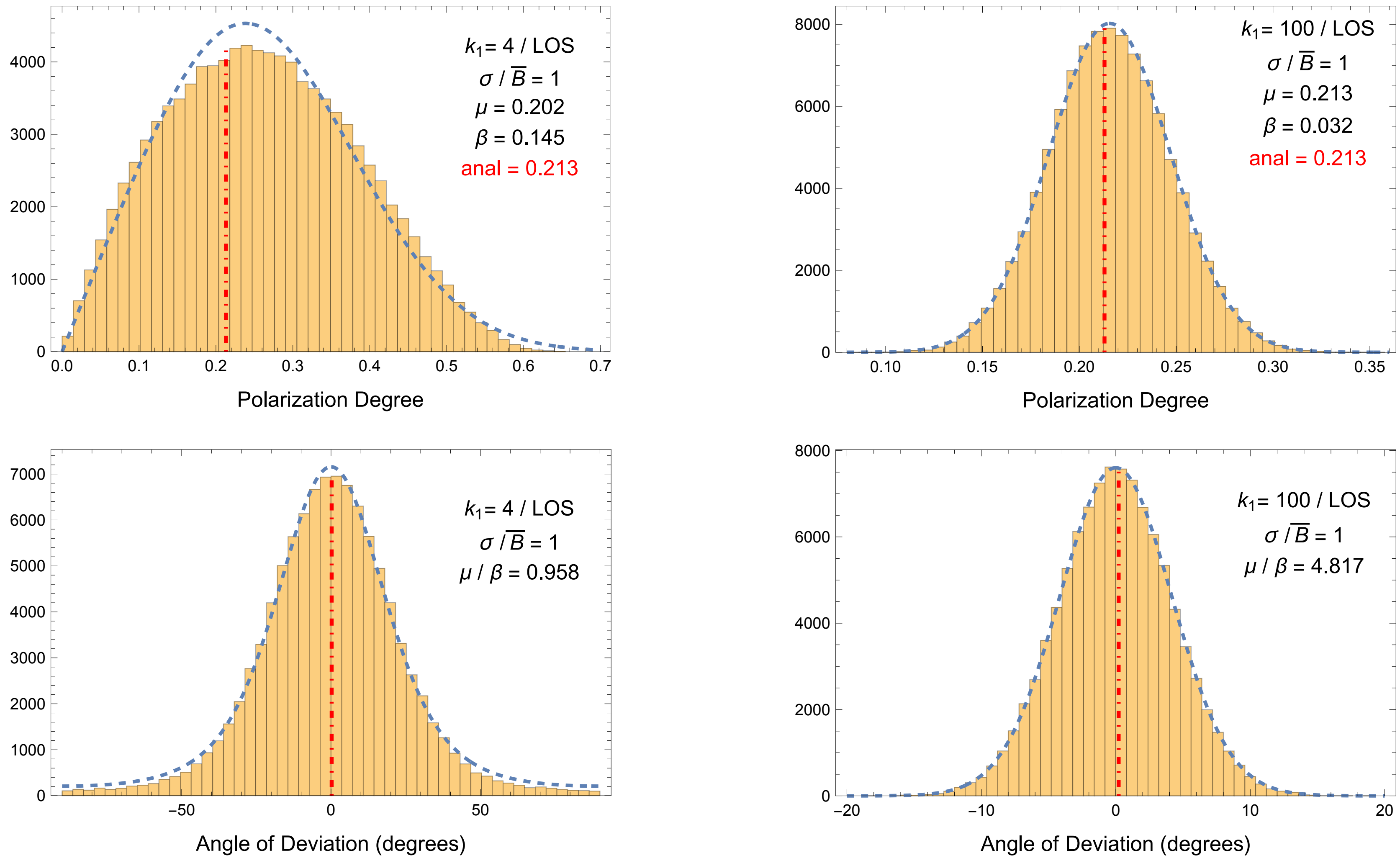
Isotropic turbulence, with $s = 2$ and $\sigma/\bar{B} = 1.0$ (100,000 cases)



Gaussianity approached, and lower dispersion, when $1/k_1 \ll$ line of sight

CONCLUSIONS

- Synchrotron polarization (in radio + X rays) is a powerful diagnostic tool. Needed to investigate (local / global) anisotropies and the level of a magnetic field turbulence.

- But it must be "handled with care".
  Importance of a combined modelling, spectrum + polarization.

- Comparison of synchrotron emission in different spectral ranges.
  BUT emission must come from the same region
  (…or at least from regions with similar magnetic fields).

- IXPE opened a new observation window. Future missions will be very useful to better characterize the physical conditions in SNRs.