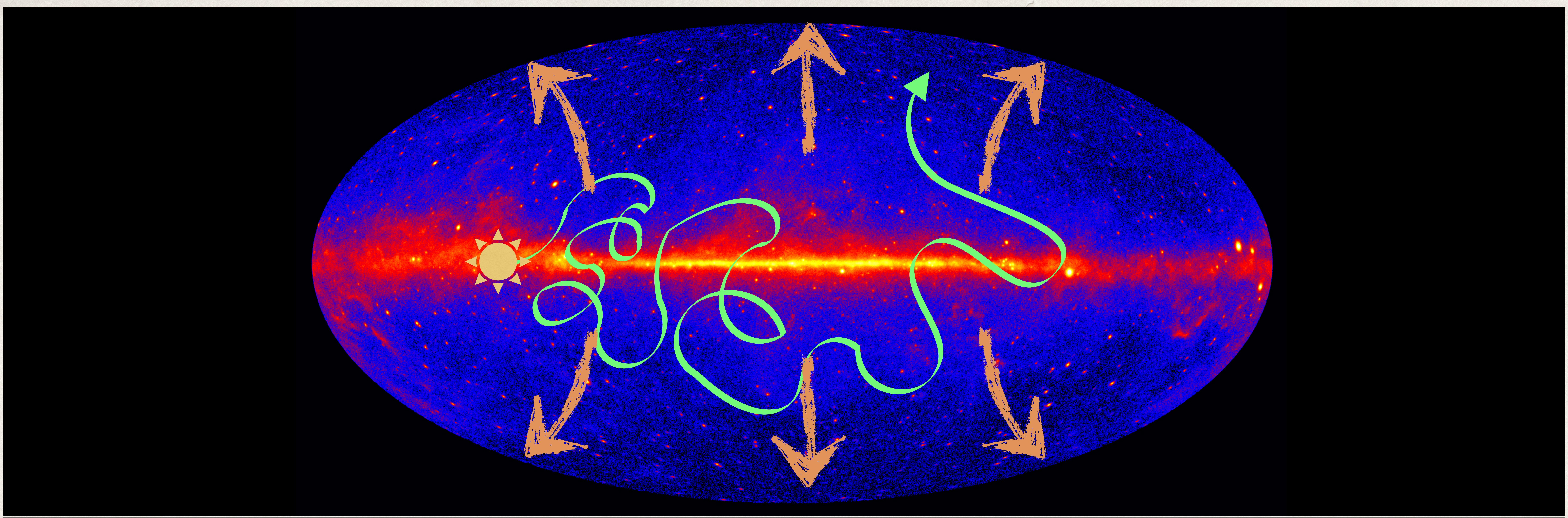

Acceleration and escape of electrons from SNRs

Based on Morlino & Celli, 2021, arXiv:2106.06488

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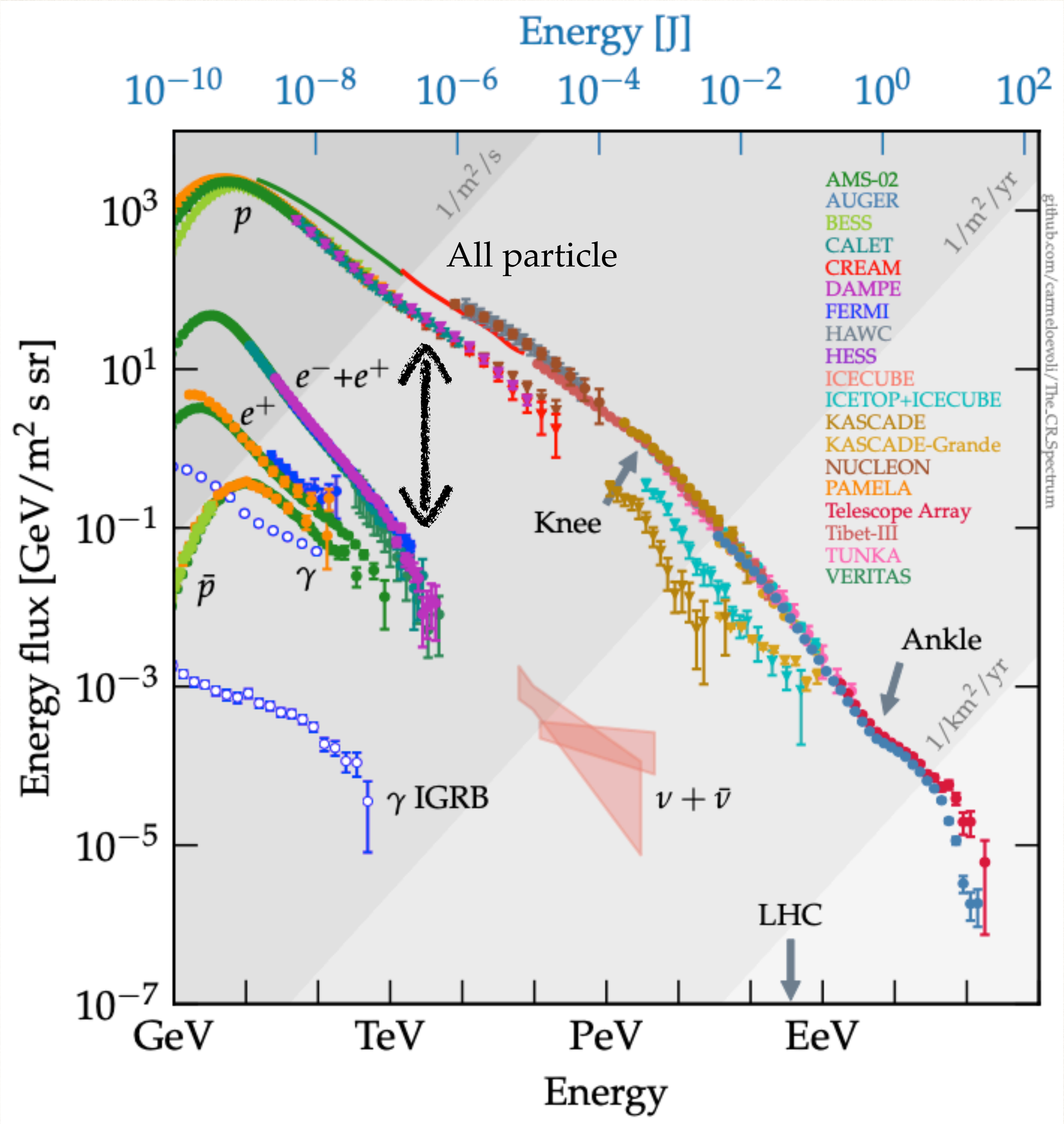
SUPERNOVA REMNANT III
AN ODYSSEY IN SPACE AFTER STELLAR DEATH
9-15 JUNE 2024, CHANIA, CRETE

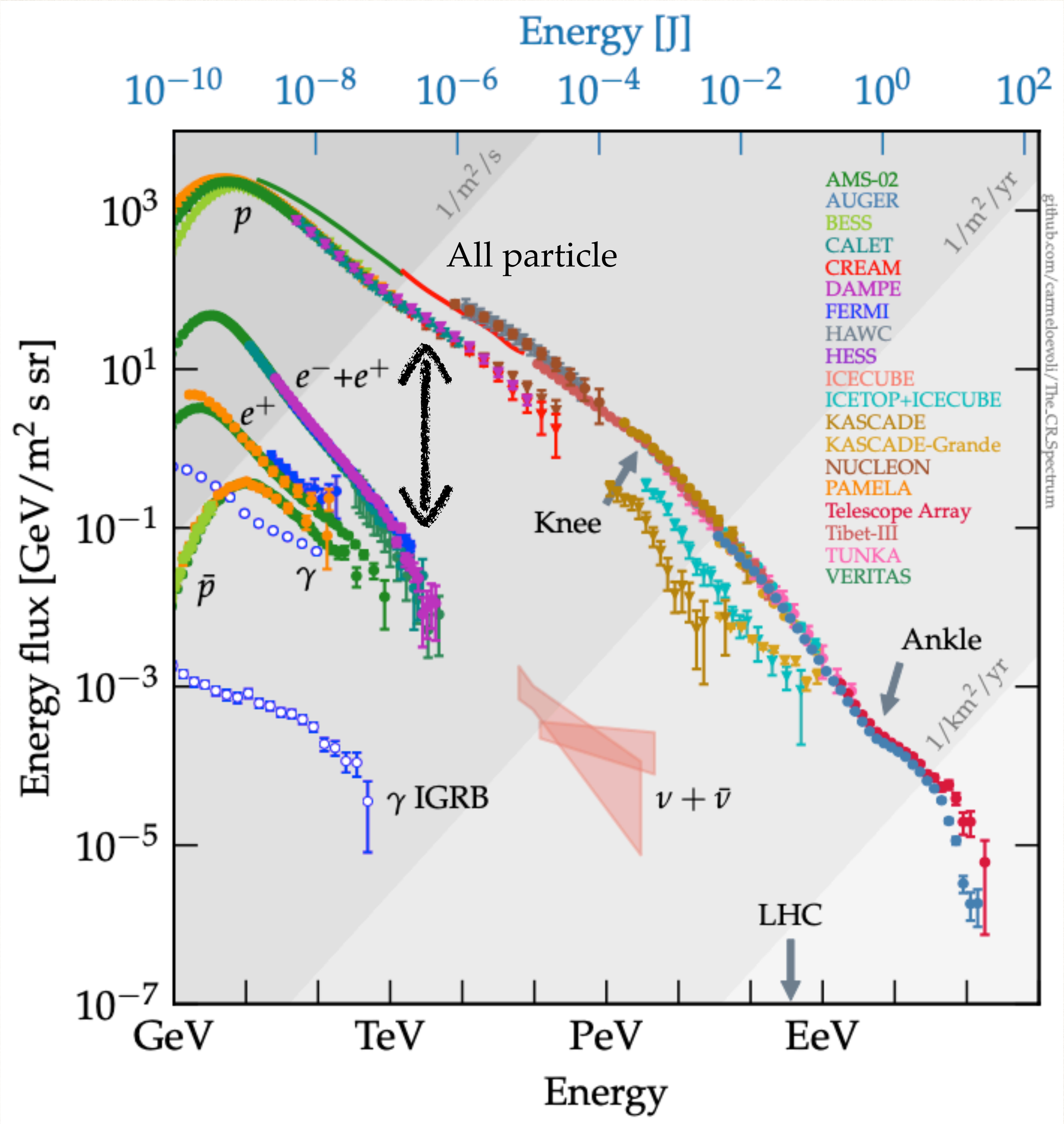
The CR spectrum as seen today



- Understanding CR physics requires to explain the spectra of each single component, including leptons

- Why electron and proton spectra are so different?
  - different sources?
  - different acceleration mechanisms?
  - same acceleration mechanism with different properties?
  - different propagation (energy losses)?

The CR spectrum as seen today



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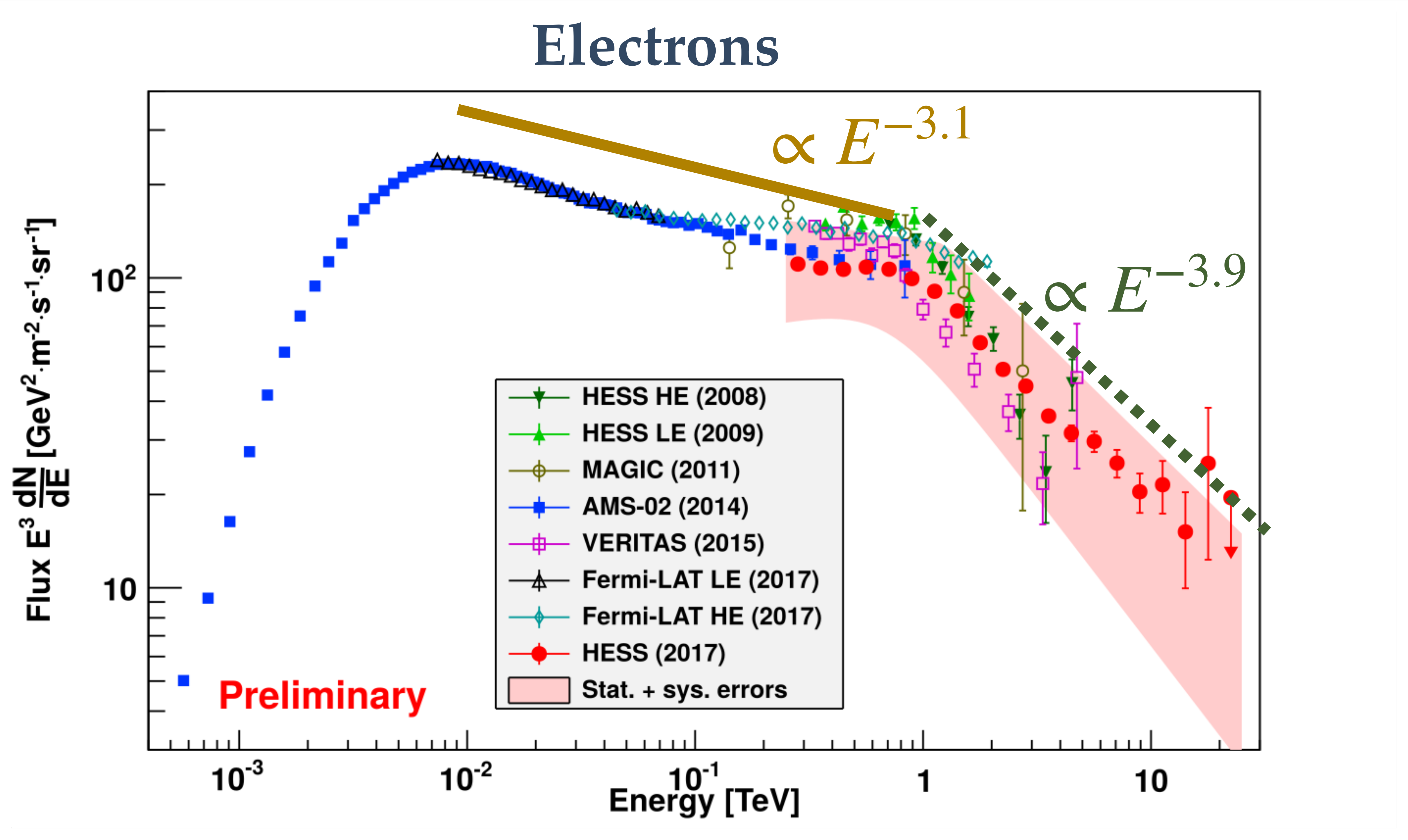
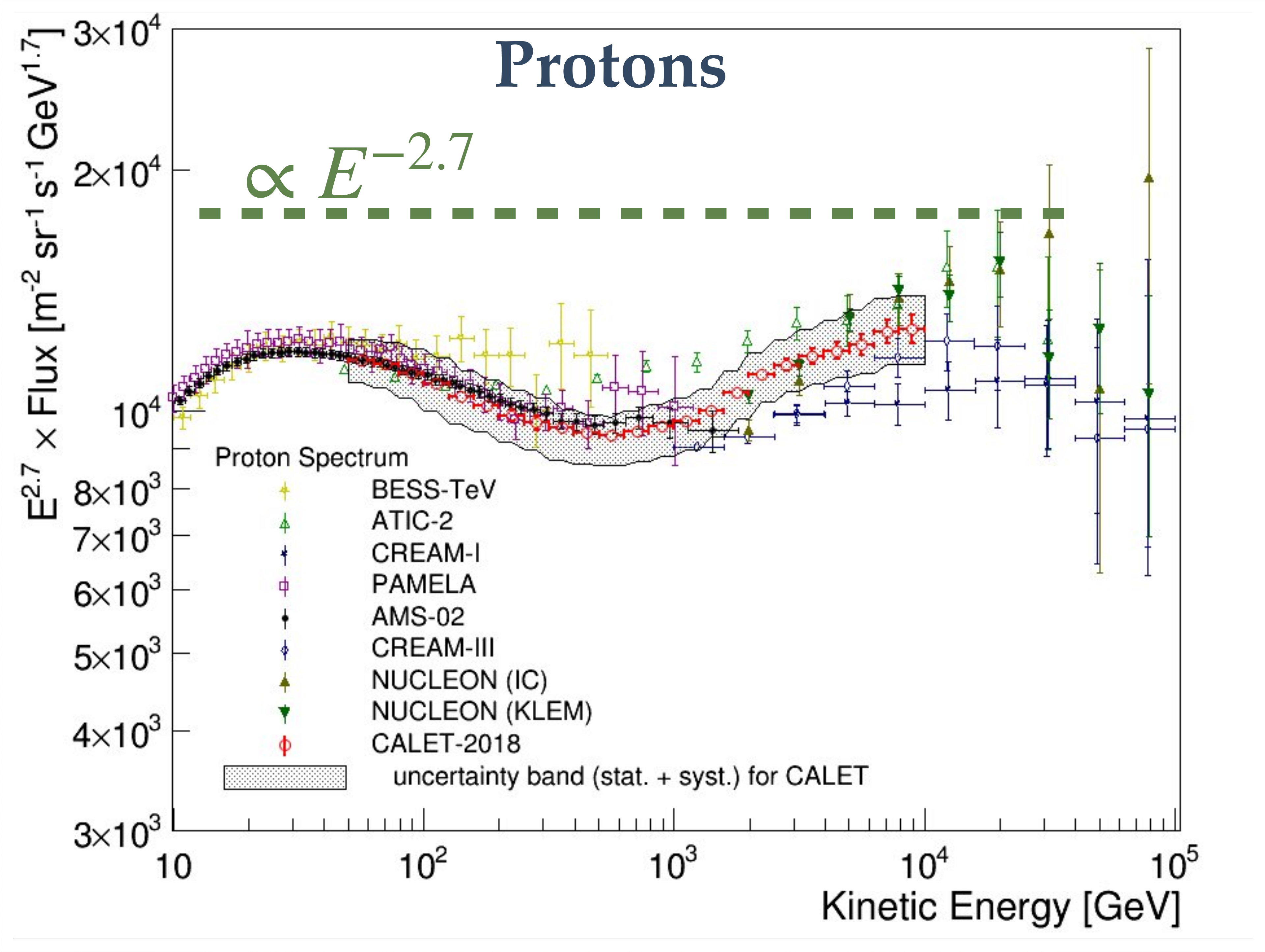
CR Electron vs. CR Proton spectrum





Slope difference
with respect to protons:

$$\Delta s_{ep} \simeq 0.4 \quad \text{for} \quad 10\,\text{GeV} \lesssim E \lesssim 1\,\text{TeV}$$
$$\Delta s_{ep} \simeq 1.2 \quad \text{for} \quad 1\,\text{TeV} \lesssim E \lesssim 20\,\text{TeV}$$

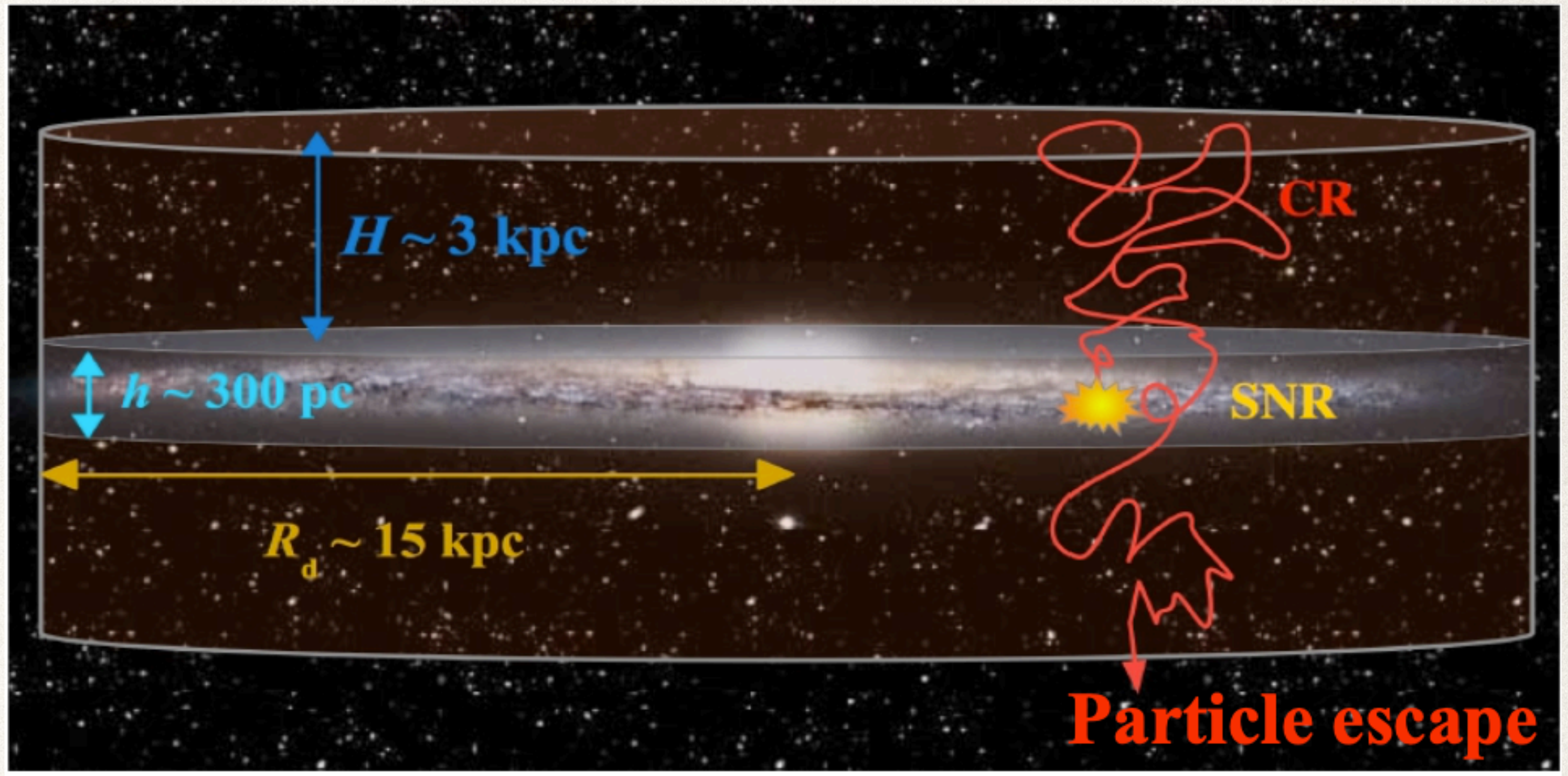
Does propagation effects may explain the difference?

The magnetic halo model for CR propagation

Ginzburg & Syrovatskii (1964); Berezinskii et al. (1980)

The basic picture

- CRs diffuse in a magnetic halo larger than the Galactic disk
- CRs freely escape from the halo boundary (half thickness H)
- The diffusion coefficient $D(E)$ is assumed constant everywhere in the halo
- The escaping time from the halo is $\tau_{\rm esc} = \dfrac{H^2}{2D(E)}$



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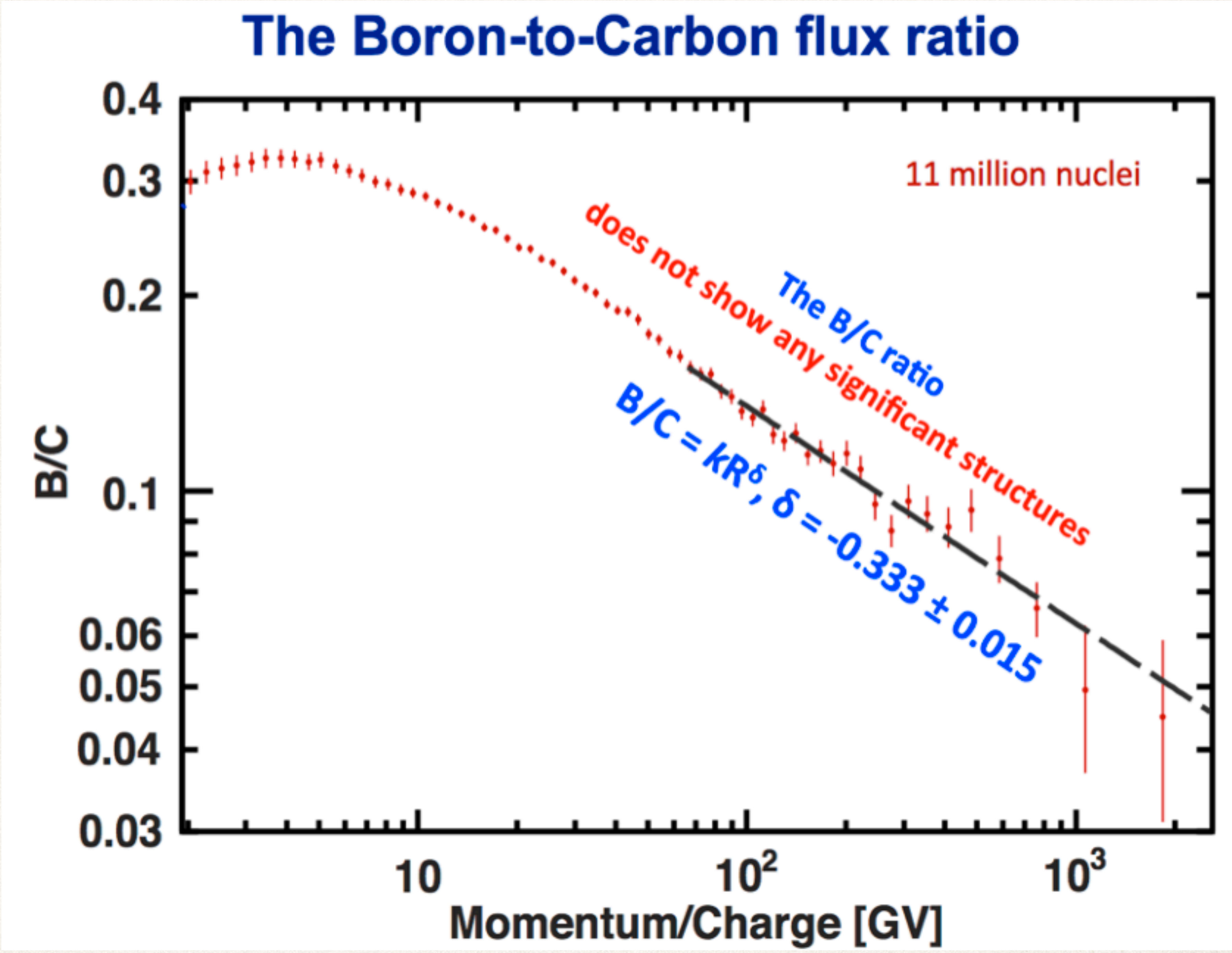

- From B/C only $\dfrac{H}{D}$ can be determined

$$\begin{cases} N_C \propto Q_{C,\rm inj}\tau_{\rm esc} \\ N_B \propto \bar{n}_{\rm gas}\sigma_{C\to B}N_C\tau_{\rm esc} \\ \bar{n}_{\rm gas} \simeq n_{\rm disk}h/H \end{cases} \Rightarrow \frac{N_B}{N_C} = n_{\rm disk}\sigma_{C\to B}\frac{H}{D(E)}$$

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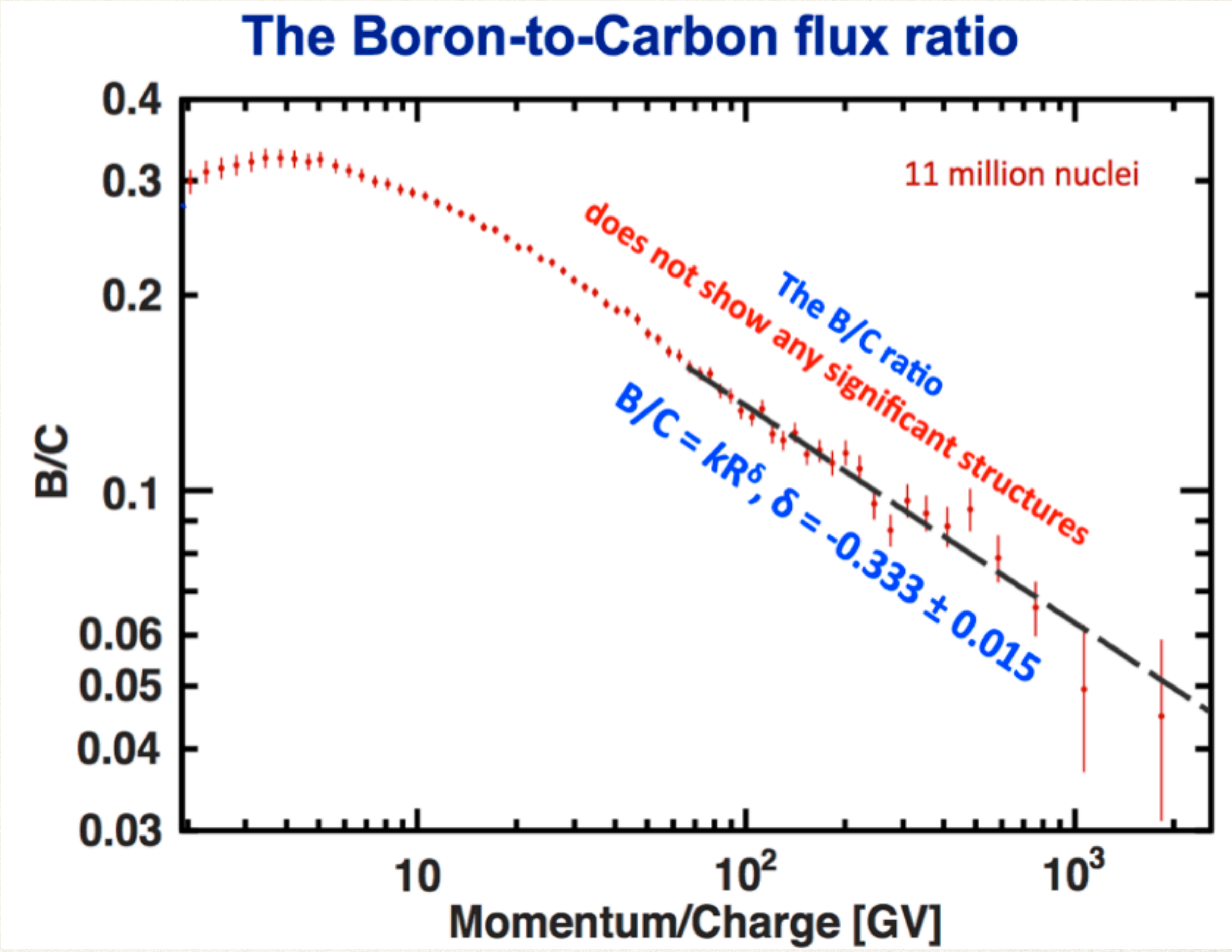

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- From B/C only $\dfrac{H}{D}$ can be determined
- **BUT the description of electron propagation requires the knowledge of both *H* and *D***

Determining the residence time from unstable nuclei

Evoli, GM, Blasi, Aloisio, PRD 2020

<u>Unstable secondary nuclei can be used to constrain the residence time of CRs inside the Galaxy, breaking the degeneracy between *H* and *D*.</u>

- ^{10}Be is especially useful because of its long half-life of 1.39 My.

$$^{10}Be \rightarrow {}^{10}B$$

- Decay reduces the flux of Be at small rigidities such that

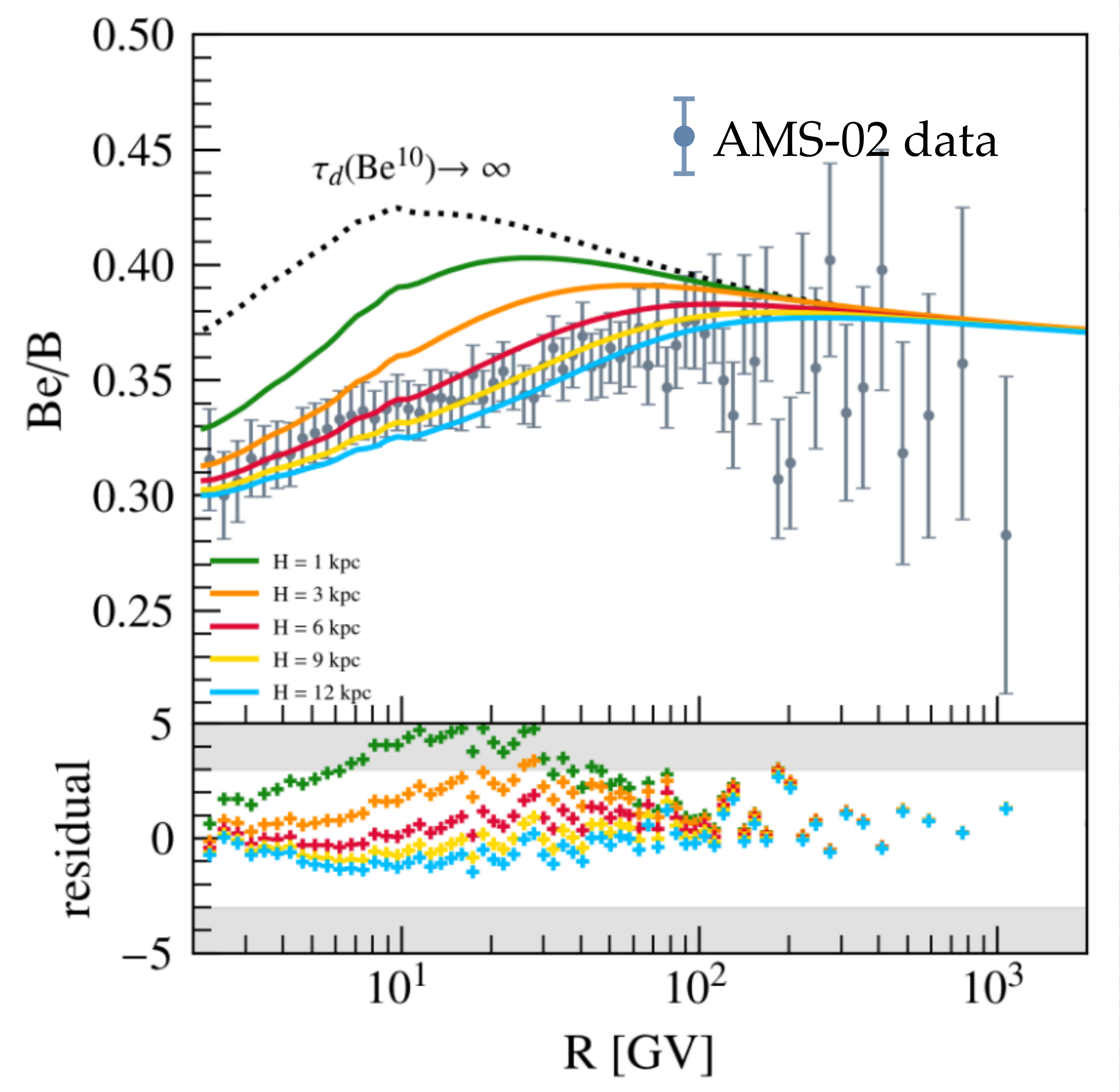
$$\gamma\,\tau_{\rm decay} \lesssim \tau_{\rm esc}(R) = \frac{H^2}{2D(R)} \;\Rightarrow\; R \lesssim 100\,\mathrm{GV}$$

- AMS-02 measurements of **Be/B** are compatible with the standard picture of CR diffusion in a halo with thickness

$$\boxed{H \gtrsim 5\ \mathrm{kpc}}$$

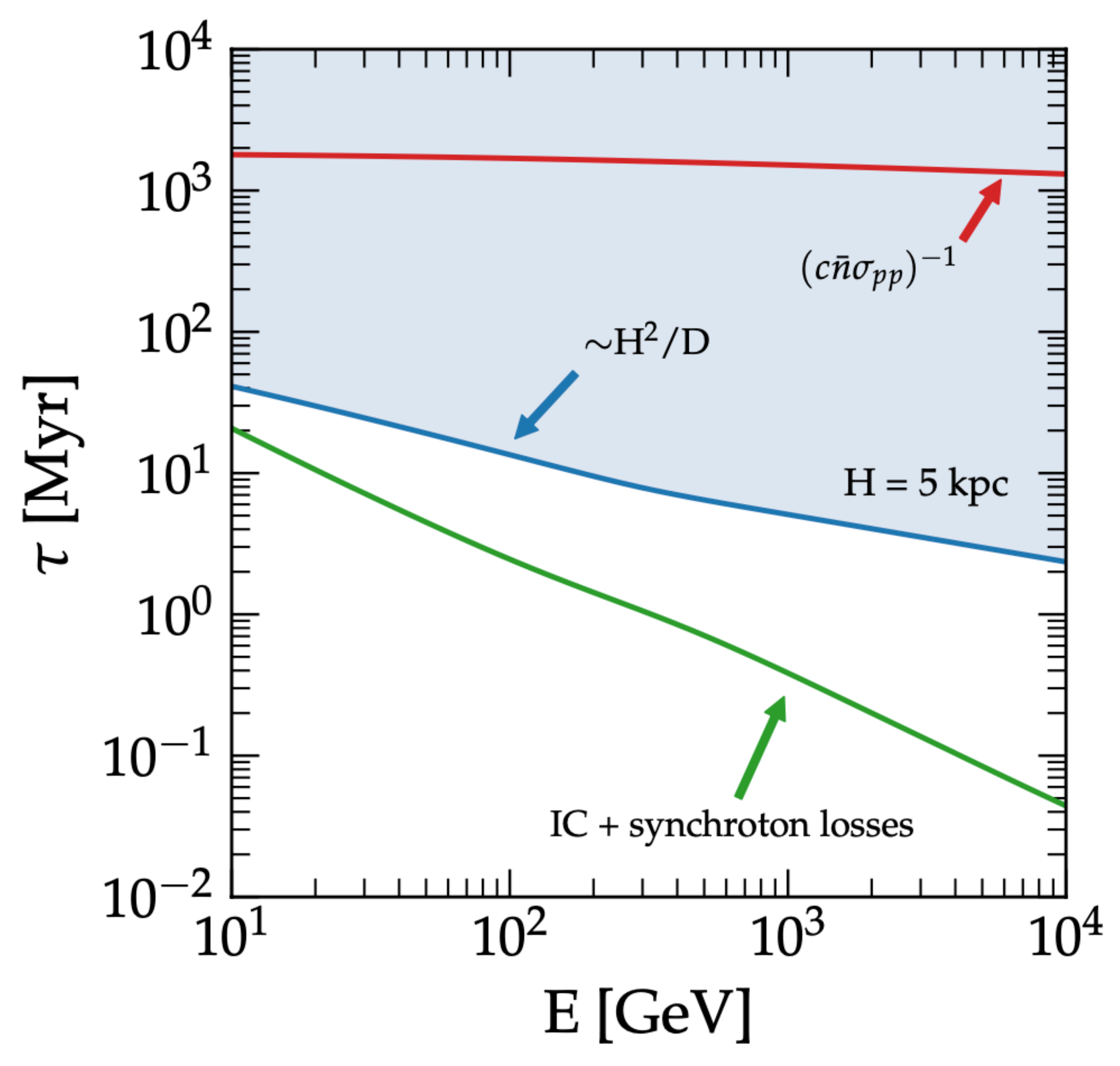
- A different analysis [Weinrich+ A&A 2020] of same data gives $H = 5^{+3}_{-2}$ kpc

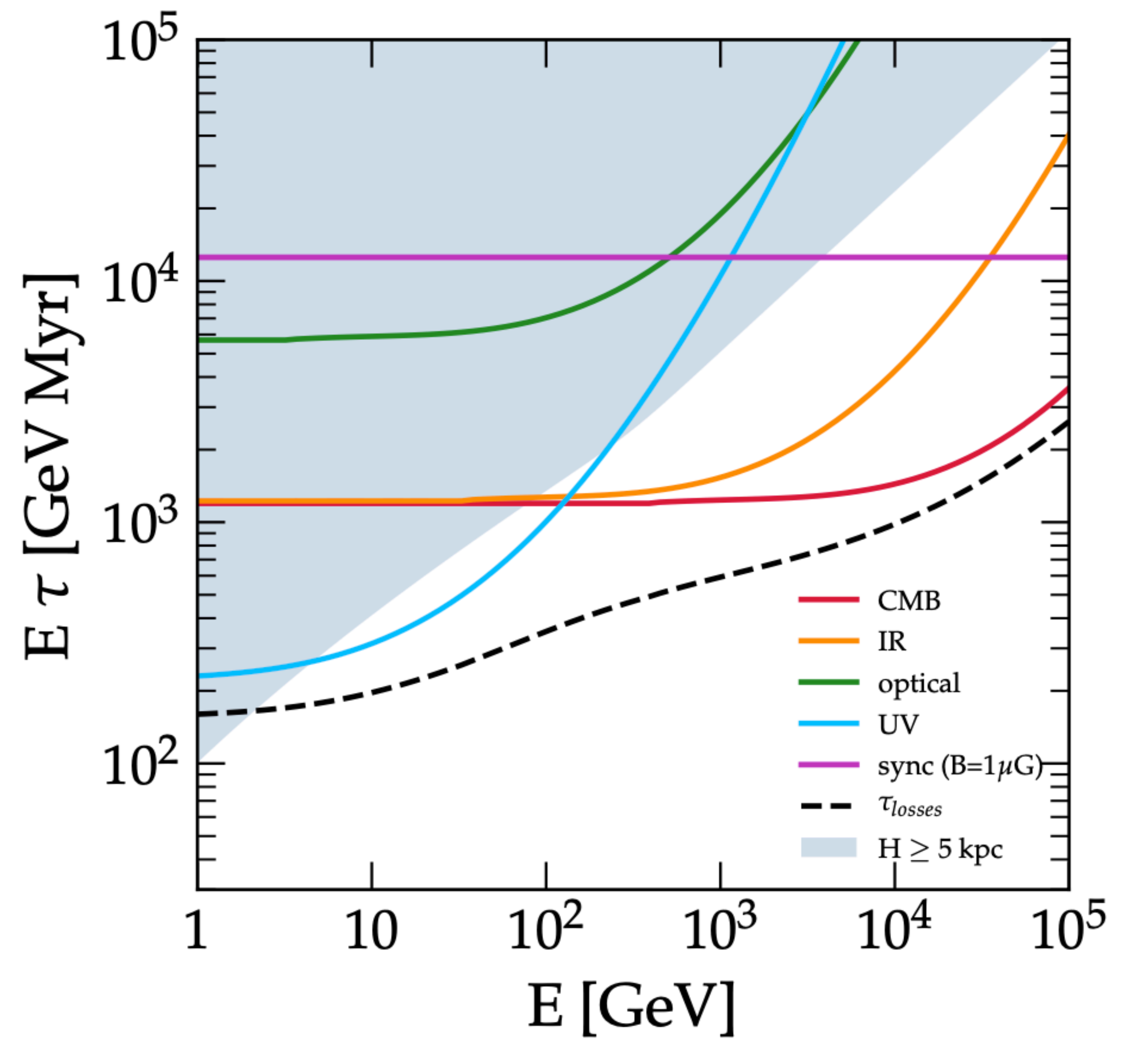
- Error mainly due to uncertainties in spallation cross section



Nuclei and lepton propagation timescales

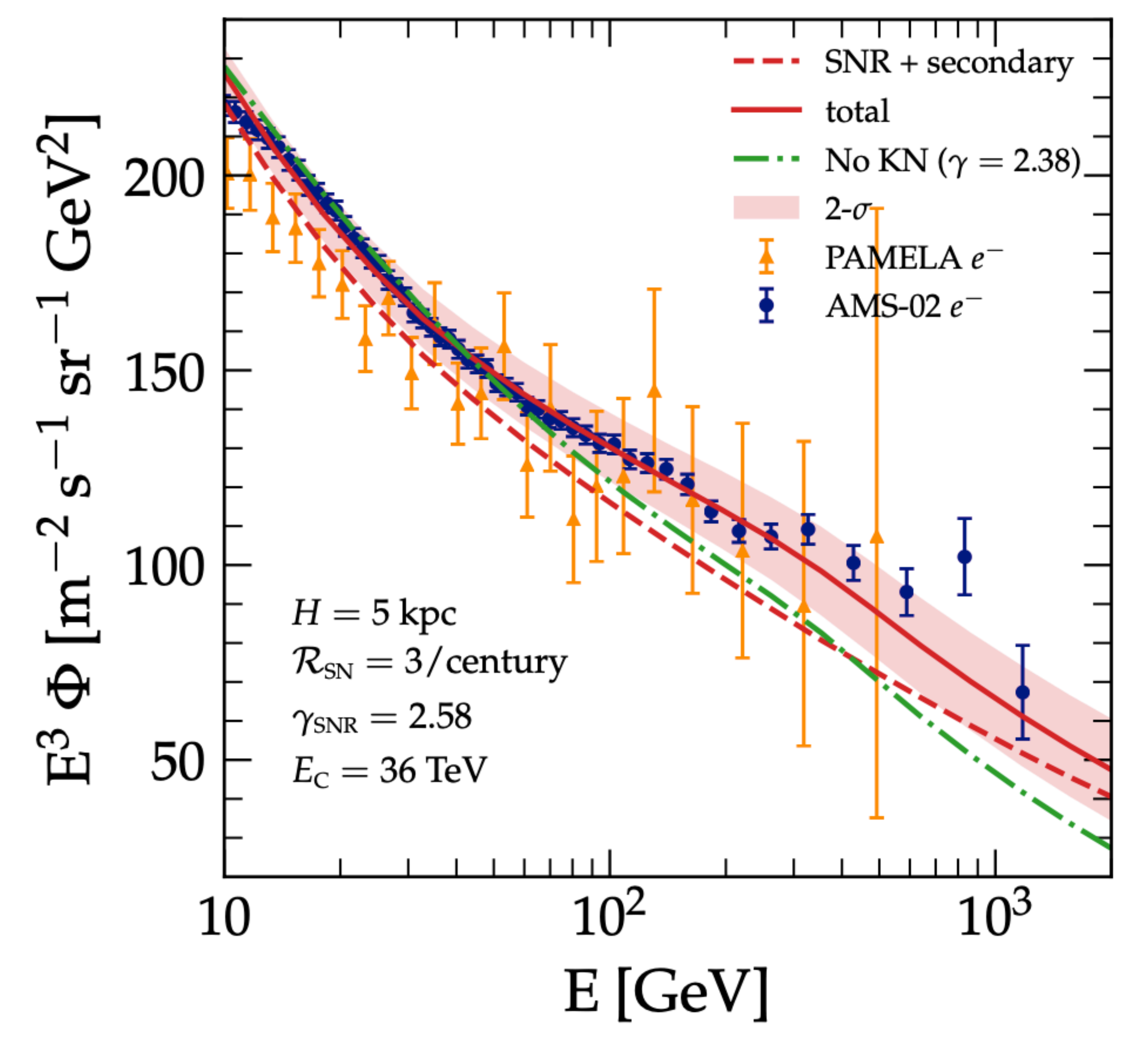
Evoli, Amato, Blasi & Aloisio (2021)





- Leptons lose their energy mainly by IC scattering with interstellar radiation field
- Milky Way is an inefficient calorimeter for nuclei but a perfect calorimeter for leptons

Nuclei and lepton propagation timescales



- After accounting for propagation and losses in the Galaxy, the slope difference between electron and protons is still large
- Electrons need to be injected by sources with:

$$Q_e(E) \sim \begin{cases} E^{-2.6} & \text{for } 10\,\mathrm{GeV} \lesssim E \lesssim 1\,\mathrm{TeV} \\ E^{-3.1} & \text{for } E \gtrsim 1\,\mathrm{TeV} \end{cases}$$

While for protons $Q_p(E) \sim E^{-2.3}$

Hence the slope difference at the source is

$$\Delta s_{\mathrm{ep}} \simeq 0.3 \quad \text{for } 10\,\mathrm{GeV} \lesssim E \lesssim 1\,\mathrm{TeV}$$
$$\Delta s_{\mathrm{ep}} \simeq 1.0 \quad \text{for } 1\,\mathrm{TeV} \lesssim E \lesssim 20\,\mathrm{TeV}$$

Evoli, Amato, Blasi, Aloisio. 2021, PRD, 103
Di Mauro, Donato, Manconi, 2020, arXiv:2010.13825

The SNR paradigm for the origin of CRs

- SNRs are thought to be the main factories of CR nuclei through *diffusive shock acceleration* (DSA)

- We assume that SNR are also responsible for the bulk production of *both* protons and electrons

- But DSA works in the same way for all particles with two exceptions:

  - ➡ energy losses during acceleration
  - ➡ energy losses during the storage time in the SNR
  - ➡ possible different injection into DSA (because of the different mass)

We will explore these mechanisms

Model for particle acceleration

SNR evolution (a very simple model…)

- We account only for SNR expanding into uniform medium (type-Ia like SNe)
    - ➡ Expansion following Truelove & McKee(1999)

Particle acceleration

- **Proton maximum energy** decreasing in time: $p_{\max,0}(t) = p_M \left(t/t_{\rm Sed}\right)^{-\delta}$ if $t < t_{\rm Sed}$
    - ➡ Self generated magnetic field due to **streaming instability** determine the magnetic field strength
        - ➡ Magnetic field determines **electron maximum energy** : $t_{\rm acc}\left(p_{\max,e}\right) = \min[\tau_{\rm loss}(\delta B_1), t_{\rm SNR}]$
- Particle acceleration stops at beginning of the radiative phase (as suggested by radio observations [Cioffi et al.(1988)] (not clear why, Mach number still $\gtrsim 10$)

Evolution of maximum energy at the shock

Protons

$$p_{\max,0}(t) = \begin{cases} p_M \left(t/t_{\rm Sed}\right) & \text{if } t < t_{\rm Sed} \\ p_M \left(t/t_{\rm Sed}\right)^{-\delta} & \text{if } t < t_{\rm Sed} \end{cases}$$

Escaping time: $t_{\rm esc}(p) = t_{\rm Sed} \left(p/p_M\right)^{-1/\delta}$

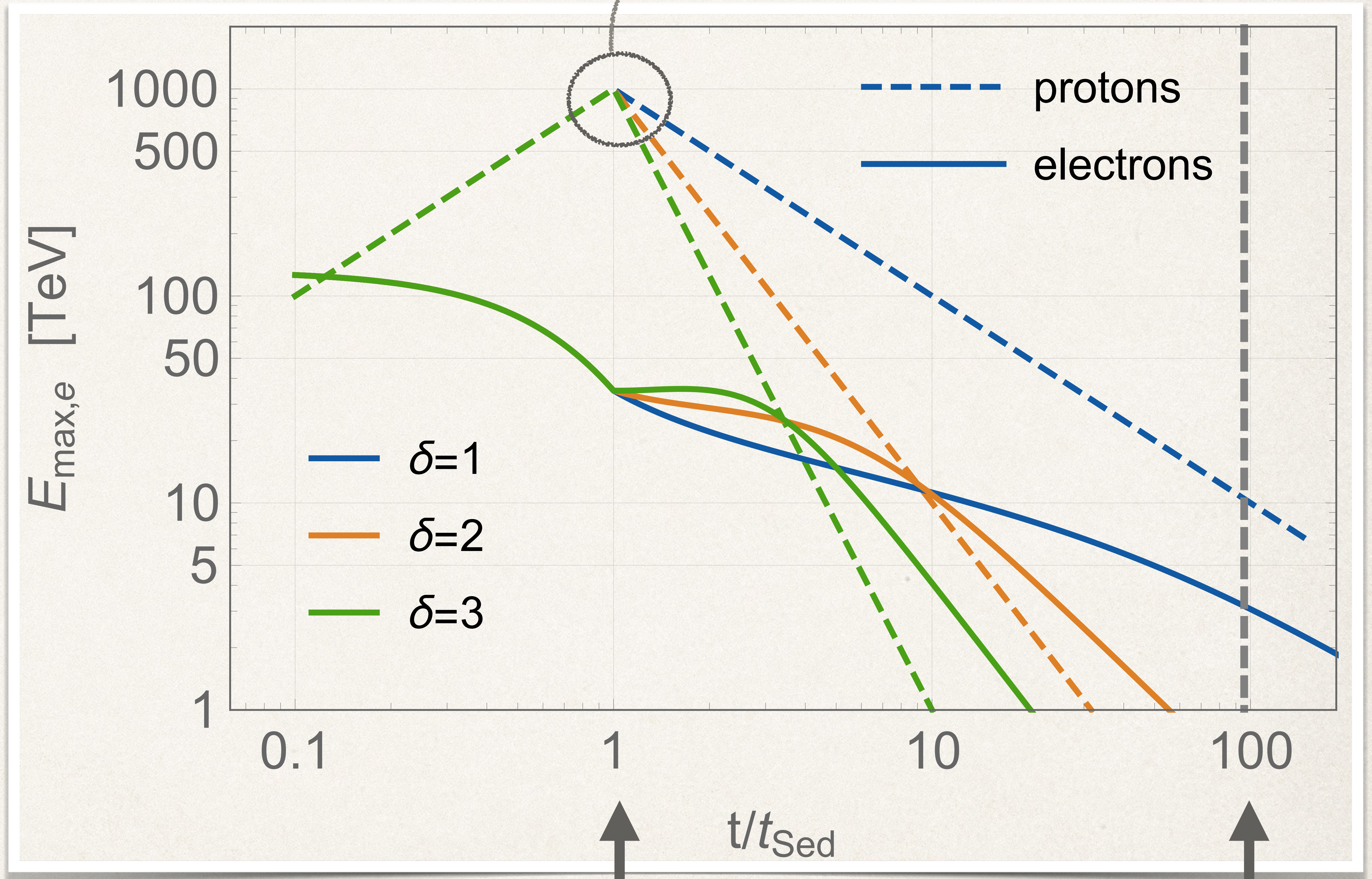
δ is a free parameter: from observations $\delta \sim 2 - 3$

Electrons

$$p_{\max,e}(t): \quad t_{\rm acc} = \min[\tau_{\rm loss}(p_{\max,e}), t_{\rm SNR}]$$

For $\delta > 1$ also electrons can escape the SNR at $t < t_{\rm SP}$

$E_M = 1\,\mathrm{PeV}$



$t_{\rm Sed} \simeq 500\,\mathrm{yr}$

$t_{\rm SP} = 95 t_{\rm Sed} \simeq 50\mathrm{kyr}$

Spectrum of escaping particles

The accelerated spectrum is: $f_{acc} \propto p^{-4}$

Particles inside the SNR start escaping when

$$p_{\text{inside}} = p_{\max,sh}(t) \rightarrow t_{\text{esc}}(p)$$

$$f_{\text{accelerated}} \neq f_{\text{released}}$$

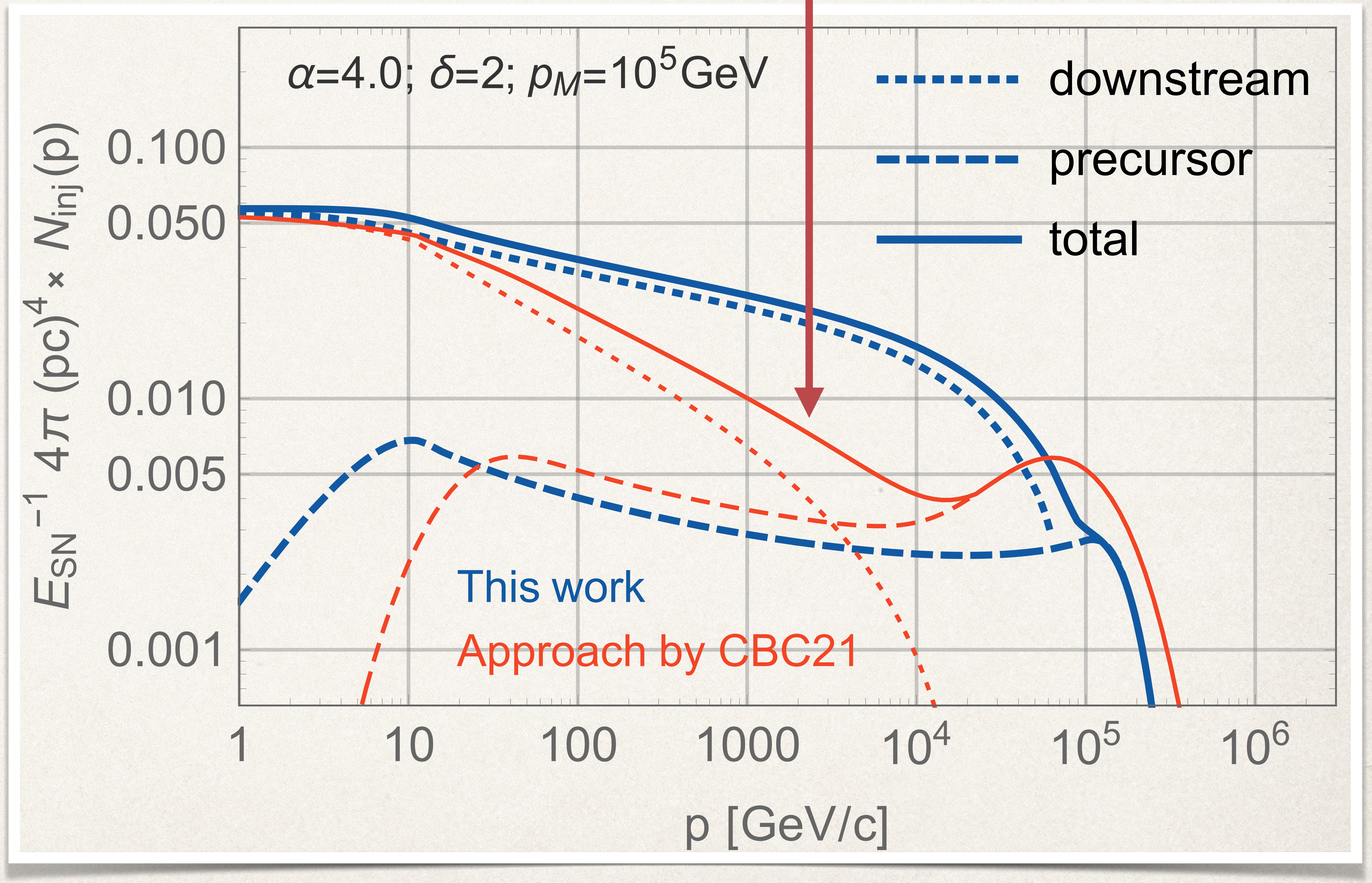
Spectrum injected into the Galaxy has two contributions:

Particles stored downstream

$$N_{\text{inj}} = \int_0^{R_{\text{esc}}(p)} 4\pi r^2 f_{\text{conf}}(p,r)dr + 4\pi R_{\text{esc}}(p)^2 \frac{D_1(p, t_{\text{esc}}(p))}{u_{\text{sh}}(t_{\text{esc}}(p))} f_0(p,(t_{\text{esc}}(p)))$$

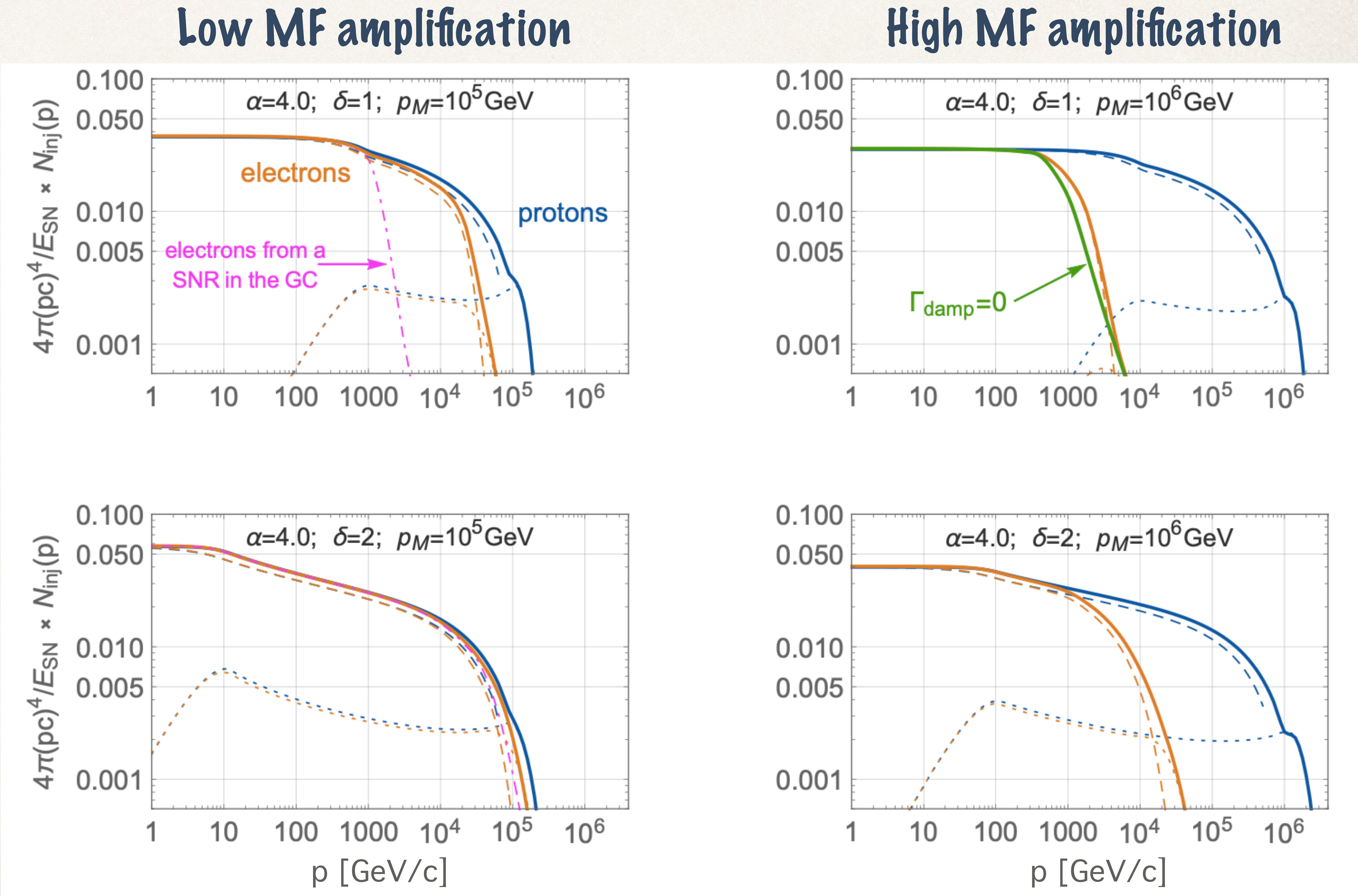
Particles escaping from the precursor

If particles escape all at the end of the Sedov phase, the final spectrum is steeper due to additional adiabatic losses



Spectrum of escaping particles:
effect of CR-amplified magnetic field

- Electron spectrum is different from the proton one if:
  - $p_M \gtrsim 1$ PeV (strong MF amplification)
  - For $\delta \lesssim 1$ (MF amplified for longer time)
  - **Differences only for $E_e \gtrsim 1$ TeV**
- Magnetic field damping does not play a significant role



Similar results also obtained by:

- Cristofari, Blasi, Caprioli, 2021, A&A, 650, A62
- Brose, Pohl, Sushch, Petruk, Kuzyo, 2020, A&A, 634, A59

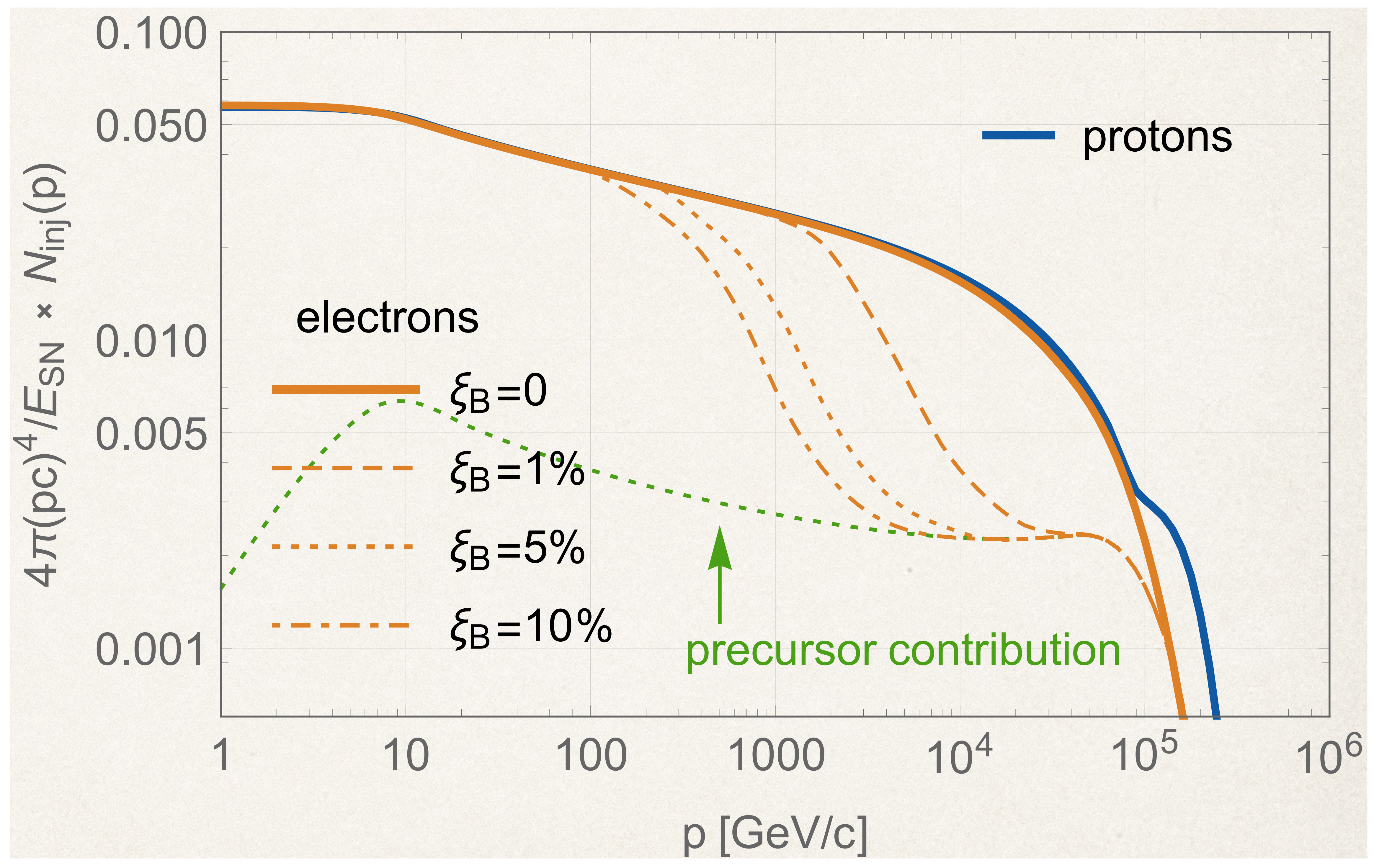
Spectrum of escaping particles:
effect of magnetic field amplified by MHD dynamo

- Magnetic field may be also amplified by turbulent MHD dynamo (i.e. Richtmeier-Meshkov instability) if the shock is propagating through a non uniform medium [see Giacalone & Jokipii, 2007]

- In this case the magnetic field is amplified only downstream and does not affect the electron maximum energy at the shock

- We assume a simple recipe: $\dfrac{B_{\rm tur}^2}{8\pi} = \xi_B \dfrac{1}{2}\rho_0 u_{\rm sh}^2$

- For $\xi_B \approx$ few percent $\Rightarrow$ steepening for $E \gtrsim 1$ TeV



Spectrum of escaping particles:
effect of magnetic field amplified by MHD dynamo

MF amplification cannot explain e/p spectral difference below ~1 TeV

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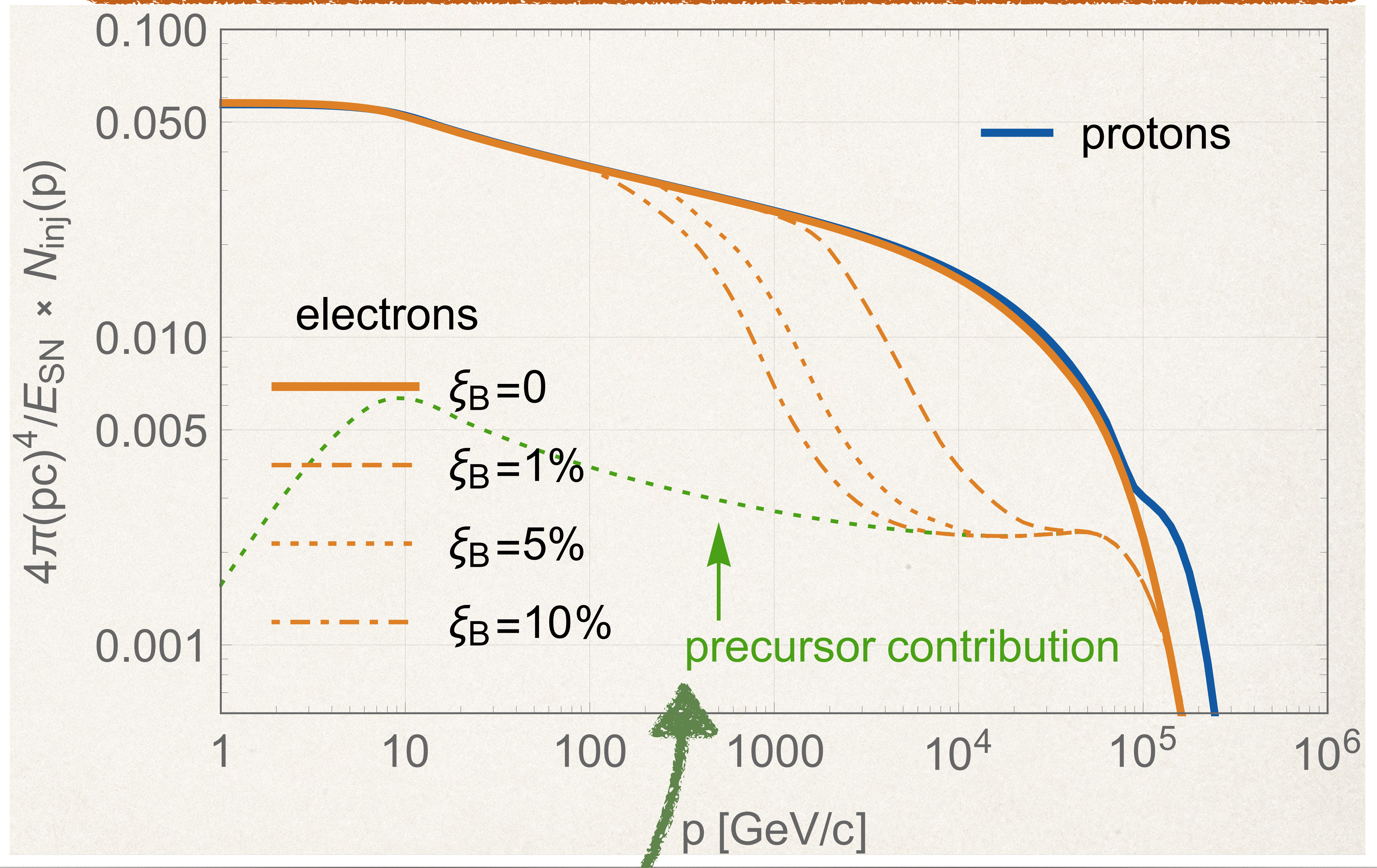


Electrons escaping from the precursor are not affected by downstream magnetic field → harder spectrum

Spectrum of escaping particles:
effect of time dependent injection

Neglecting radiative losses for electrons and assuming an instantaneous e/p spectral ratio:

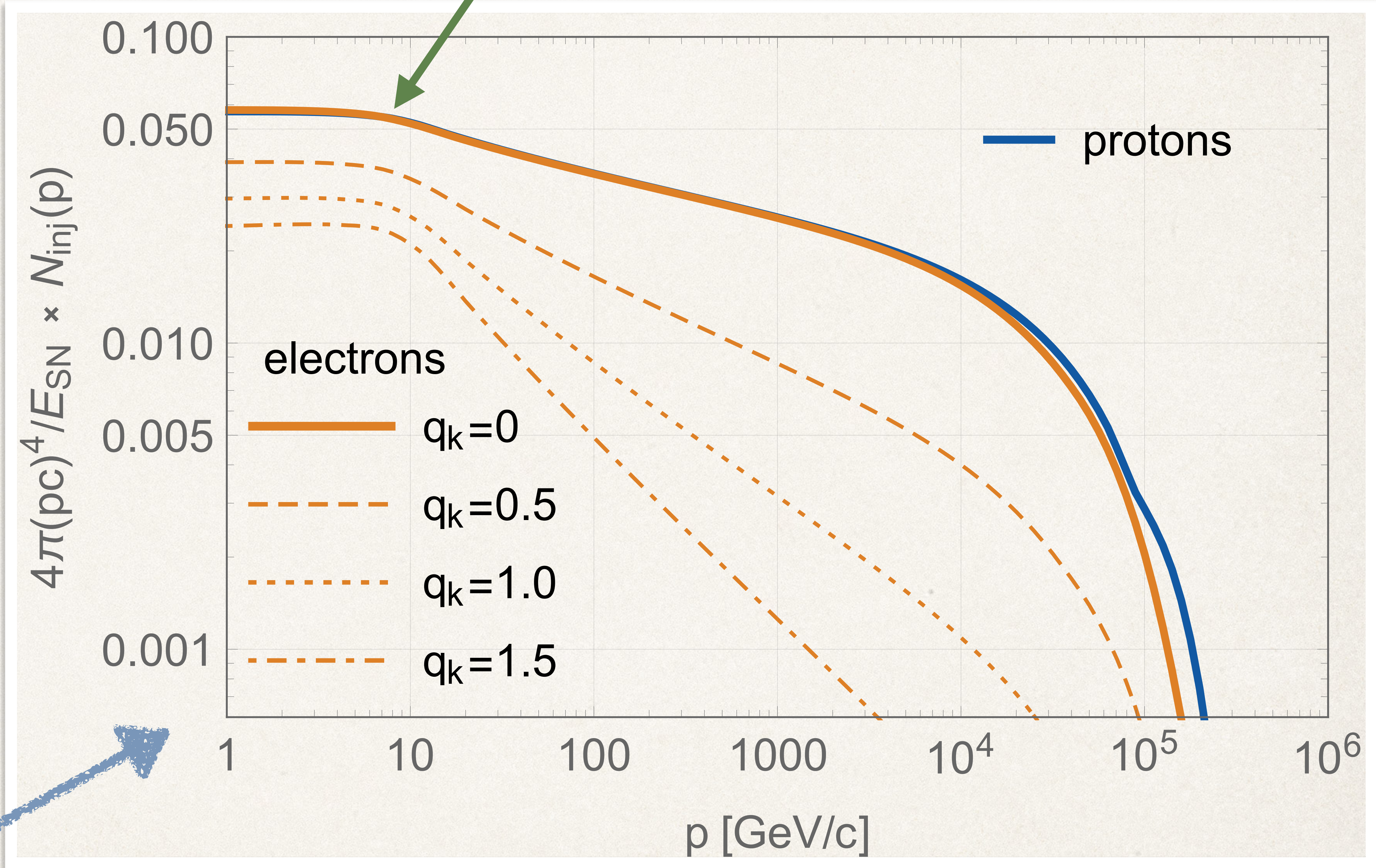
$$\frac{N_{e,\mathrm{inj}}}{N_{p,\mathrm{inj}}} \equiv K_{\mathrm{ep}}(t) = u_{\mathrm{sh}}(t)^{-q_k} \propto p^{\overbrace{-3q_k/(5\delta)}^{\Delta s_{\mathrm{ep}}}}$$

Using Sedov-Taylor evolution: $u_{\mathrm{sh}}(t_{\mathrm{esc}}) \propto t_{\mathrm{esc}}^{-3/5} \propto \left(p^{-1/\delta}\right)^{-3/5}$

$$\boxed{\Delta s_{\mathrm{ep}} = 0.3 \;\Rightarrow\; q_k = 1 \div 1.5 \;\; \text{for} \;\; \delta = 2 \div 3}$$

Prediction confirmed by full calculation

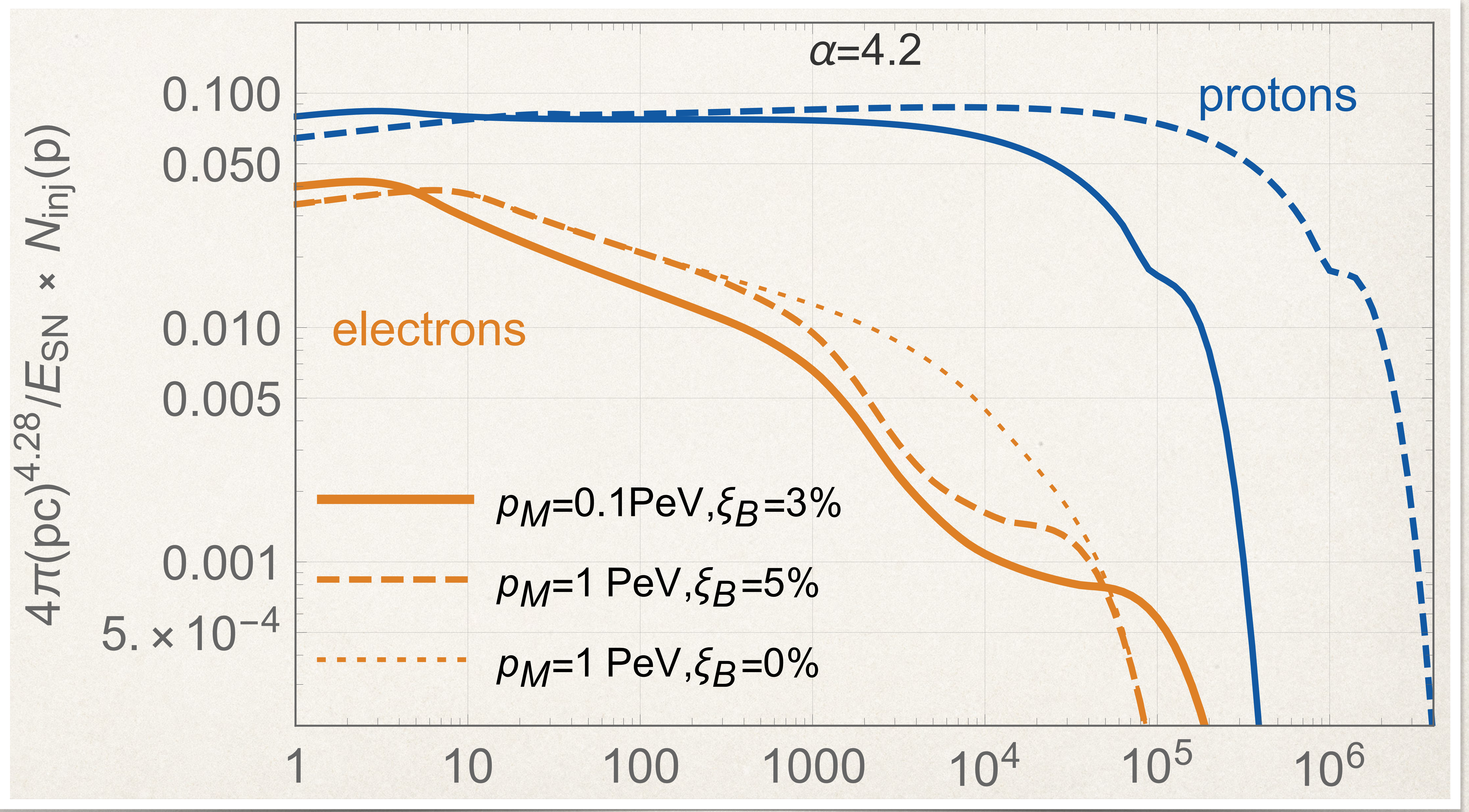
The breaking energy is the maximum energy when the acceleration stops



Accounting for all effects

Example of spectra accounting for all effects

The accelerated spectrum is: $f_{\rm acc} \propto p^{-4.2}$



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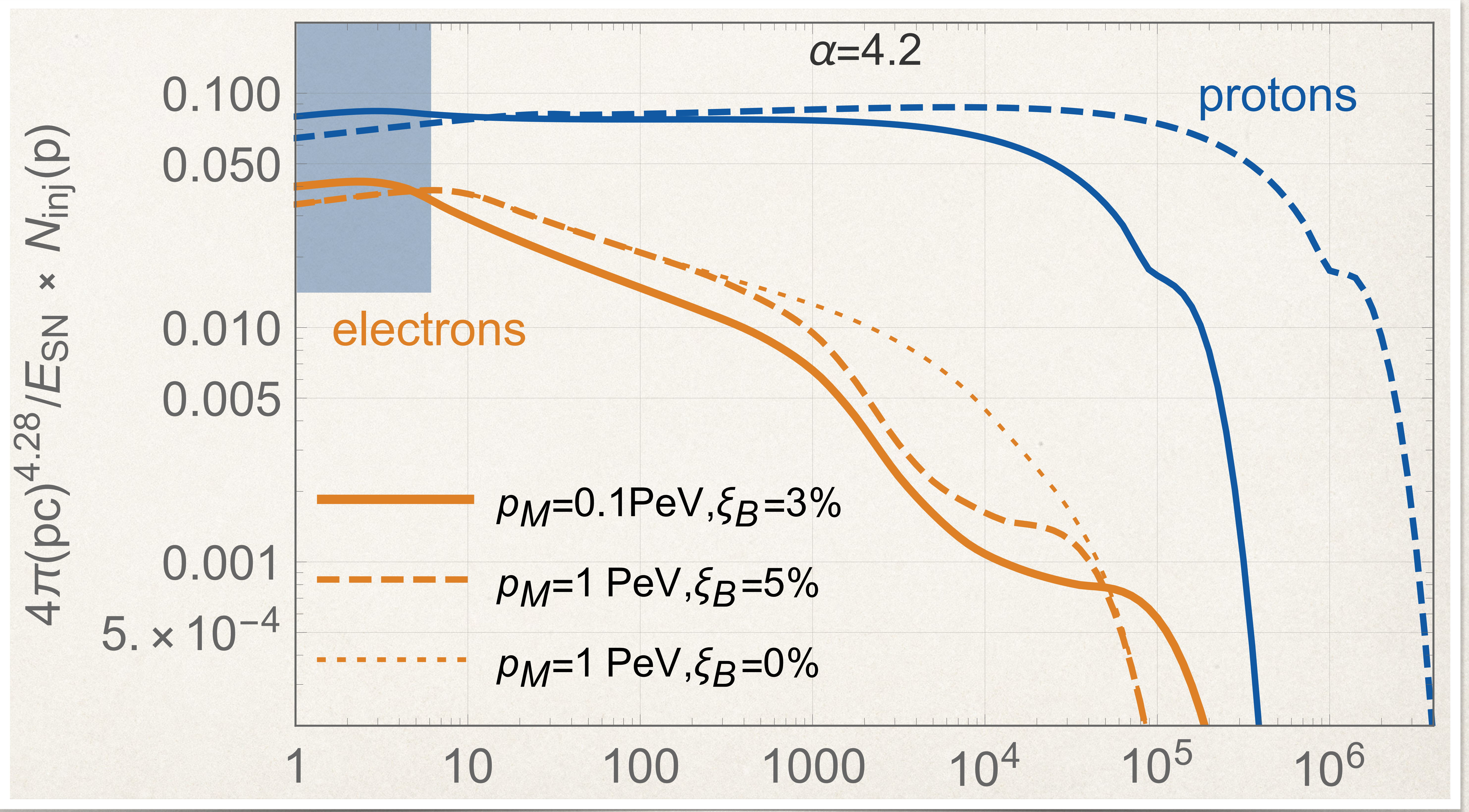
Hardening < GeV due to stop of acceleration in the radiative phase



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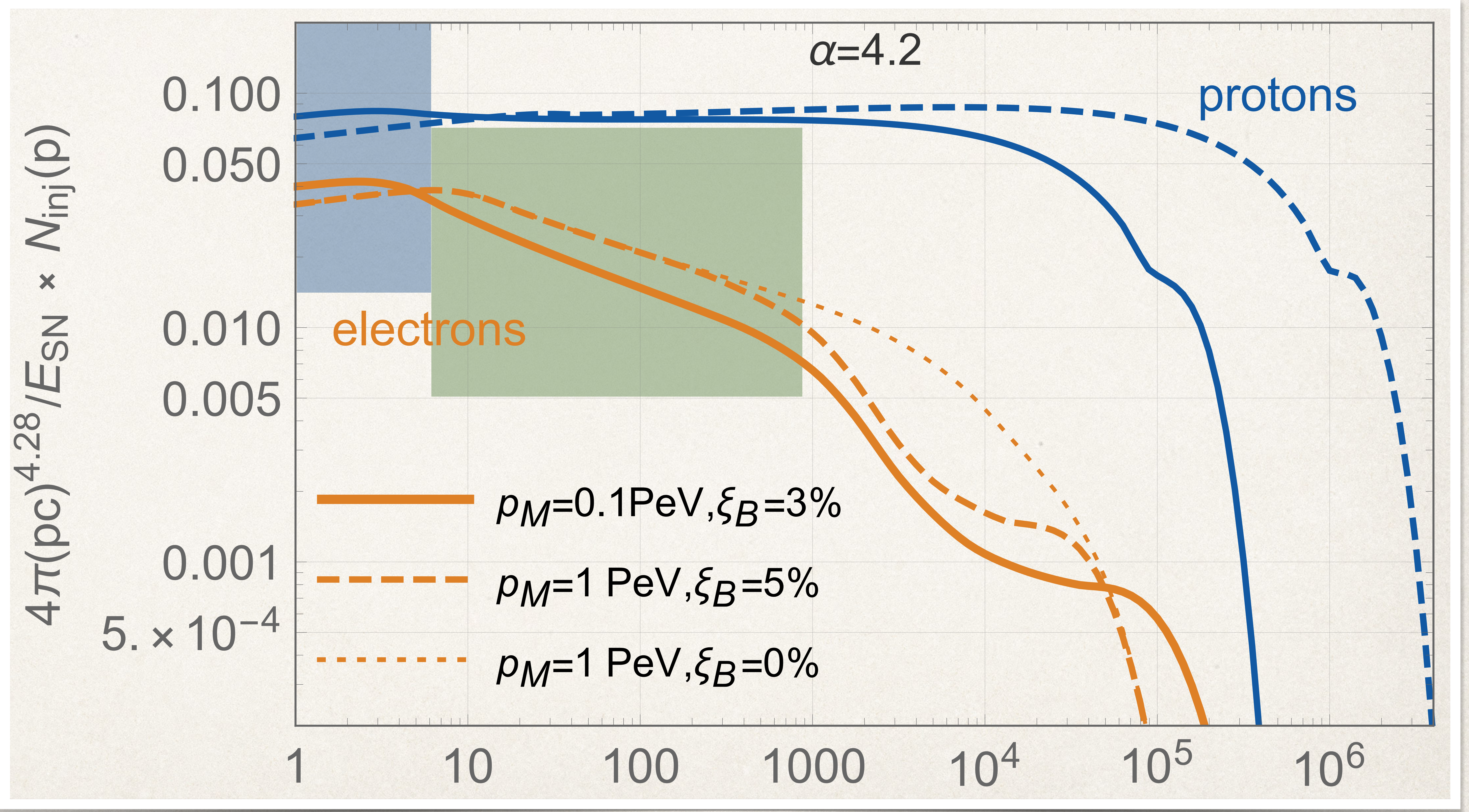
Softening due to the electron injection $K_{ep} \propto u_{sh}^{-1}$



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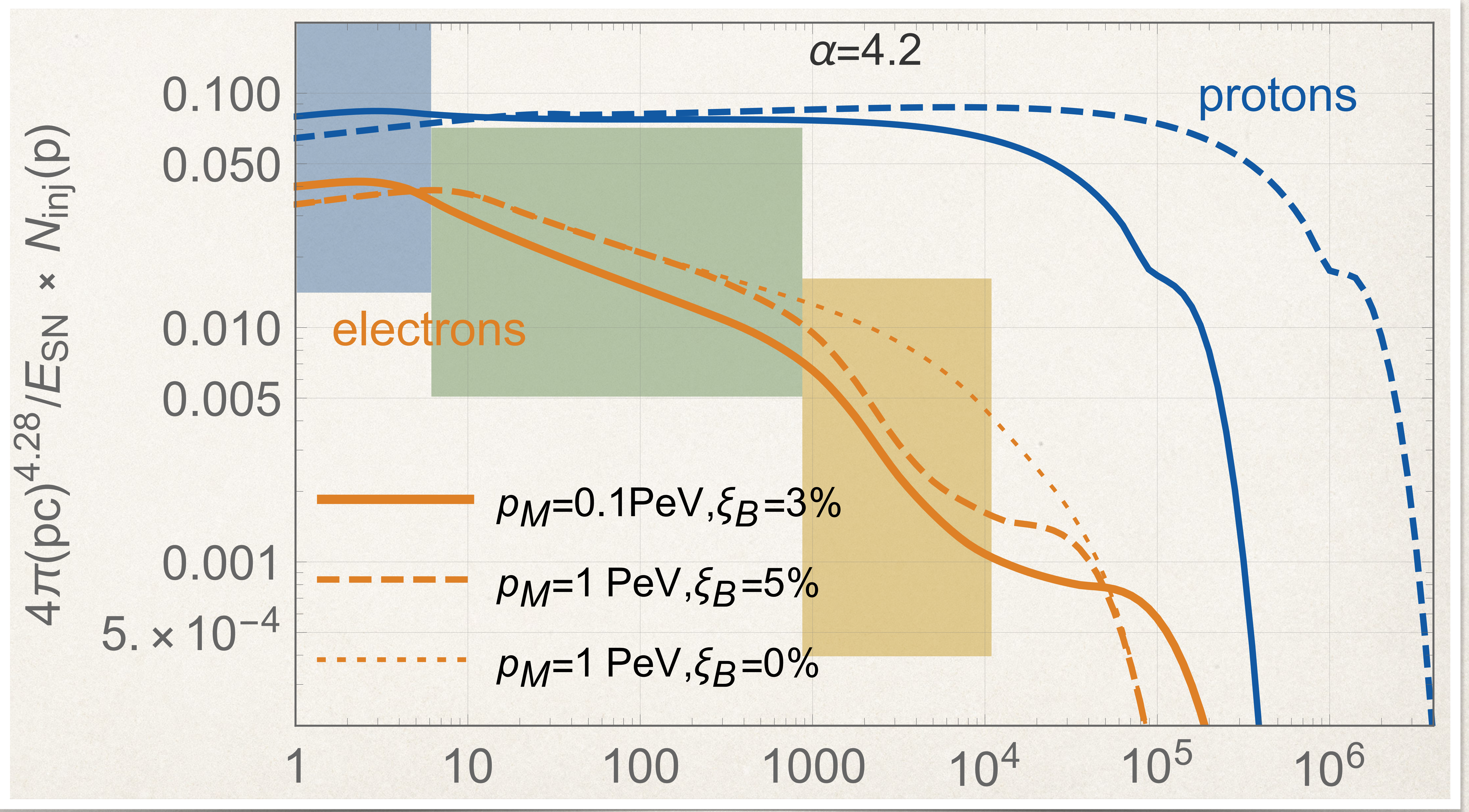
Softening due to synchrotron losses in the sources



Accounting for all effects

Hardening < GeV due to stop of acceleration in the radiative phase

Softening due to the electron injection
$K_{ep} \propto u_{sh}^{-1}$

Softening due to synchrotron losses in the sources

Hardening due to electron escape from the shock precursor (reduced losses)

Example of spectra accounting for all effects

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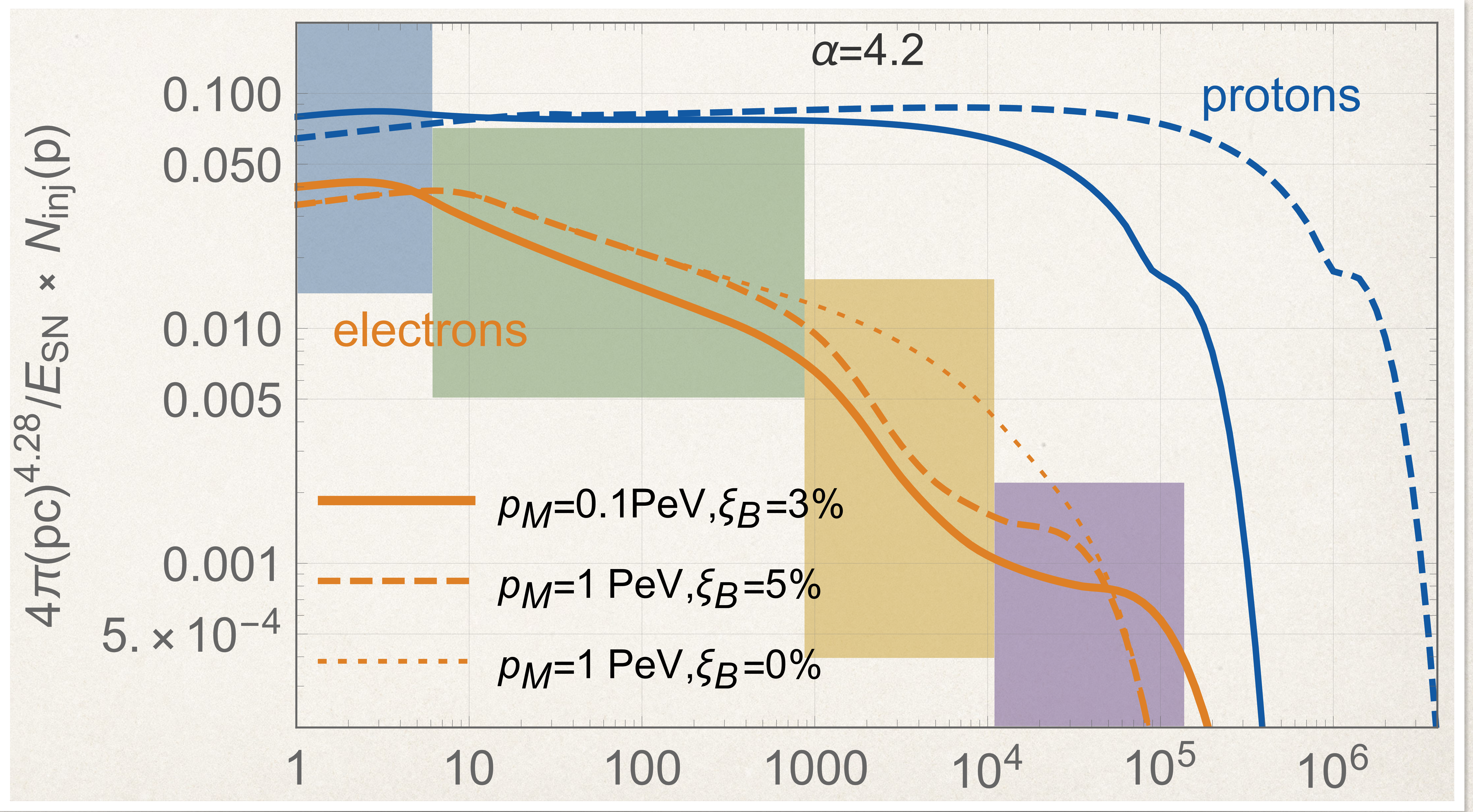


Why velocity-dependent injection for electrons?

1st evidence: observations

- The electron/proton temperature ratio immediately downstream of the shock can be inferred from Balmer emission.

- Van Adelsberg et al.(2008) inferred $T_e/T_p \propto V_{\rm sh}^{-2}$

- For plane shocks:

$$T_p \propto m_p V_{\rm sh}^2 \;\; \Rightarrow \;\; T_e \propto \text{const} \equiv \Delta E \approx 0.3\,\text{keV}$$

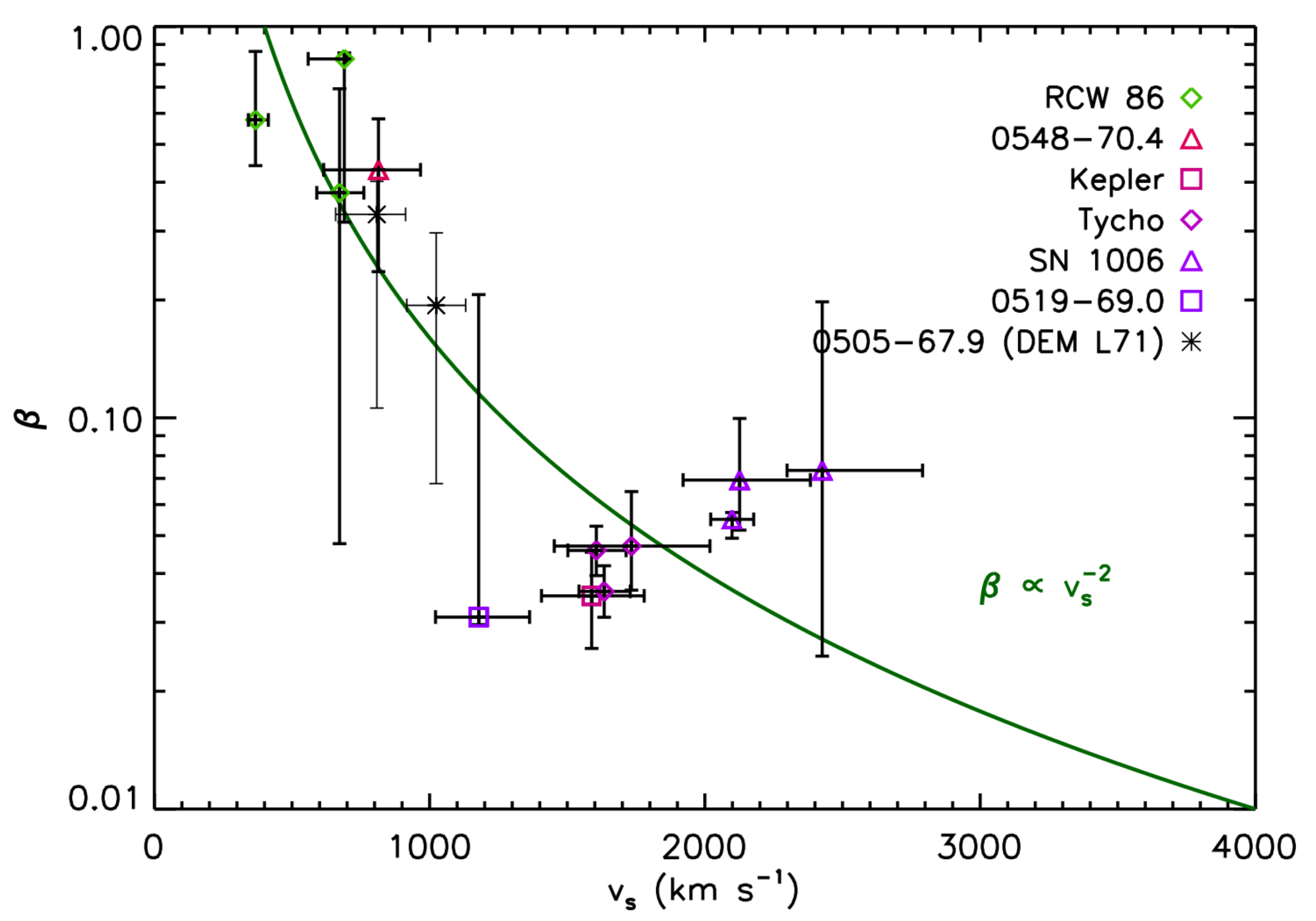
  ΔE is the energy transferred from p to e

- Caveat: this relation seems to hold mainly for $V_{\rm sh} \lesssim 2000\,\text{km/s}$

- BUT electron escape also occurs mainly for $V_{\rm sh} \lesssim 2000\,\text{km/s}$

From van Adelsberg et al.(2008)

T_e/T_p vs. V_{sh}



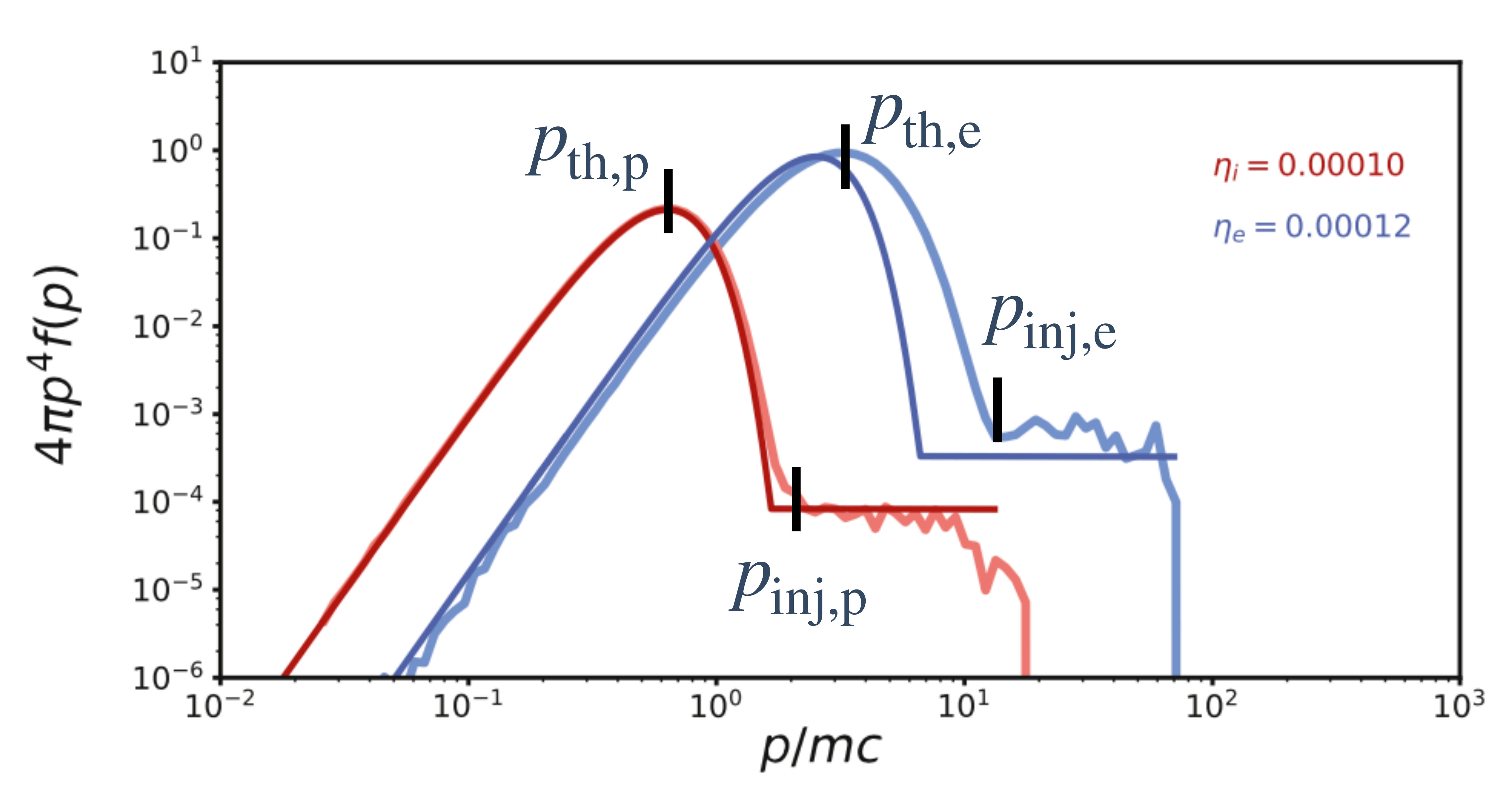
Why velocity-dependent injection for electrons?

2nd evidence: PIC simulation

- Injection of particles into DSA occurs for $p > p_{\rm inj,i} \equiv \xi_{\rm i}\, p_{\rm th,i}$ where $p_{\rm th,i} = \sqrt{2m_i K_{\rm B} T_{\rm i}}$
- The number of injected particles is $\eta_{\rm i} \propto \xi_{\rm i}^3\, e^{-\xi_i^2}$
- Arbutina & Zekovic (2021) got $\xi_e \simeq \xi_p$ from PIC simulations even if electrons with $p < p_{\rm inj,p}$ are pre-accelerated by mechanisms different from DSA
- The spectral electron/proton ratio is

$$K_{\rm ep} = \frac{\eta_e}{\eta_p}\left(\frac{p_{\rm inj,e}}{p_{\rm inj,p}}\right)^{\frac{3}{R_{\rm sub}-1}} \simeq \left(\frac{m_e}{m_p}\frac{16\Delta E}{3\, m_p u_{\rm sh}^2}\right)^{\frac{3}{2(R_{\rm sub}-1)}} \propto u_{\rm sh}^{-\frac{3}{R_{\rm sub}-1}}$$

from Arbutina & Zekovic (2021)



$$\boxed{\text{For } R_{\rm sub} \simeq 4 \;\Rightarrow\; K_{\rm ep} \sim u_{\rm sh}^{-1}}$$

Caveat: results from Arbutina & Zekovic are limited to relativistic electrons with $m_p/m_e \leq 100$

Conclusions

Context:

- CR electron spectra is steeper then the proton one by **0.4** for [10 GeV- 1 TeV] and **1.2** for [1-10 TeV]
- Losses during propagation in the Galaxy cannot account for the entire spectral differences
  - **The difference should be due to different acceleration and/or to losses inside the sources**
  - **or to different source population of electrons**

Method:

- Assuming that both protons and electrons are accelerated at SNR shocks, we investigate two mechanisms: **synchrotron losses in amplified magnetic field** and **time dependent injection**

Results:

- Synchrotron losses can steepen the spectrum only above ~TeV
- Time-dependent injection can further steepen the spectrum by ~0.3 down to ~ GeV if $K_{ep} \propto u_{sh}^{-1}$
  - Some evidences support velocity dependent injection of electrons
    - But we still lack of a full theoretical explanation

BACKUP SLIDES

The magnetic halo model

Ginzburg & Syrovatskii (1964); Berezinskii et al. (1980)

Transport equation of CR of species α:

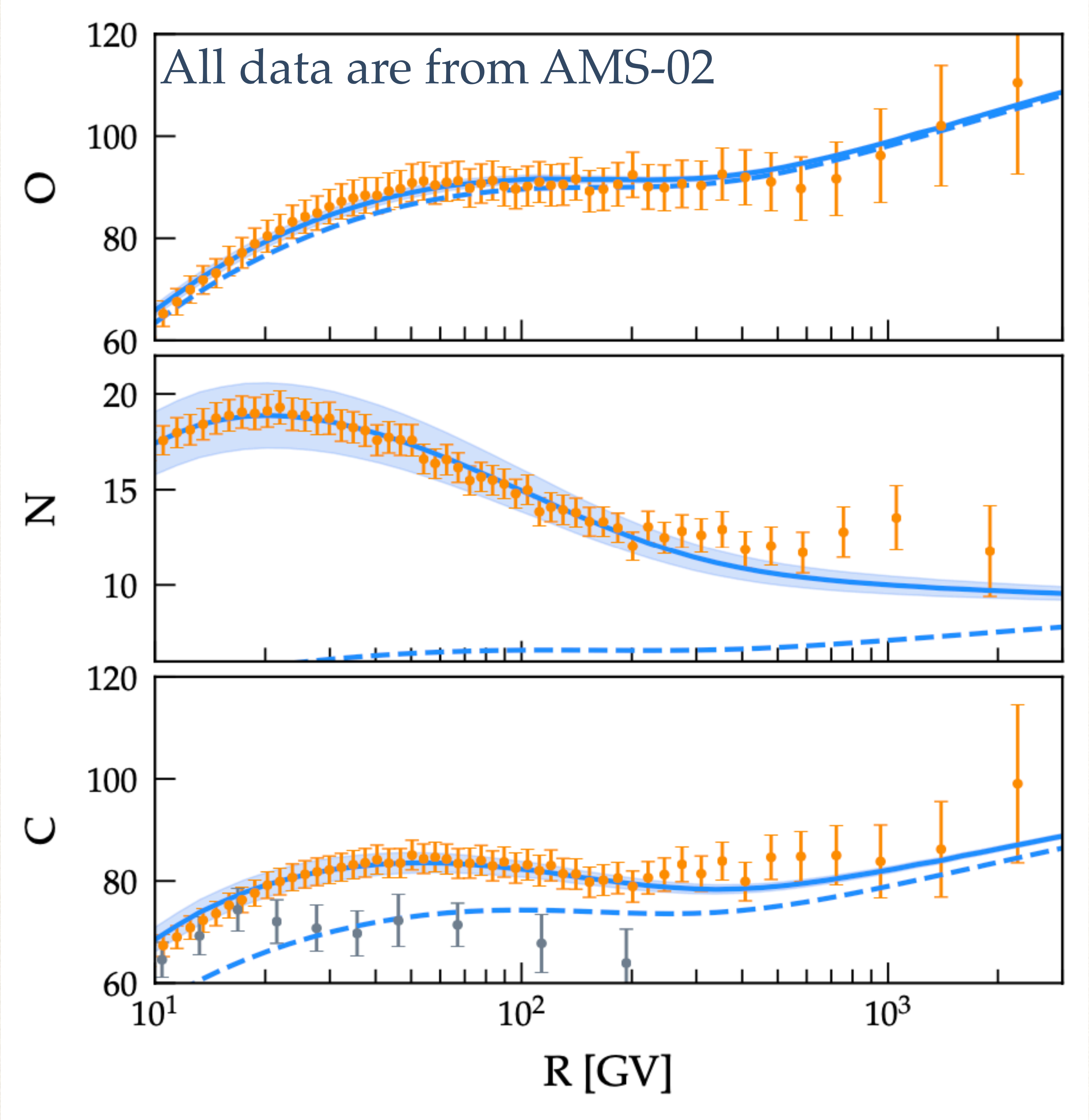
$$\cancel{\frac{\partial f_\alpha}{\partial t}} - \frac{\partial}{\partial z}\left(D\frac{\partial f_\alpha}{\partial z}\right) + u\frac{\partial f_\alpha}{\partial z} - \frac{du}{dz}\frac{p}{3}\frac{\partial f_\alpha}{\partial p} = q_{\rm SN}\delta(z) - \frac{1}{p^2}\left[p^2\dot{p}f_\alpha\right] - \frac{f_\alpha}{\tau_\alpha^{\rm in}} + \Sigma_{\alpha'>\alpha}b_{\alpha'\alpha}\frac{f}{\tau_{\alpha'}^{\rm in}}$$

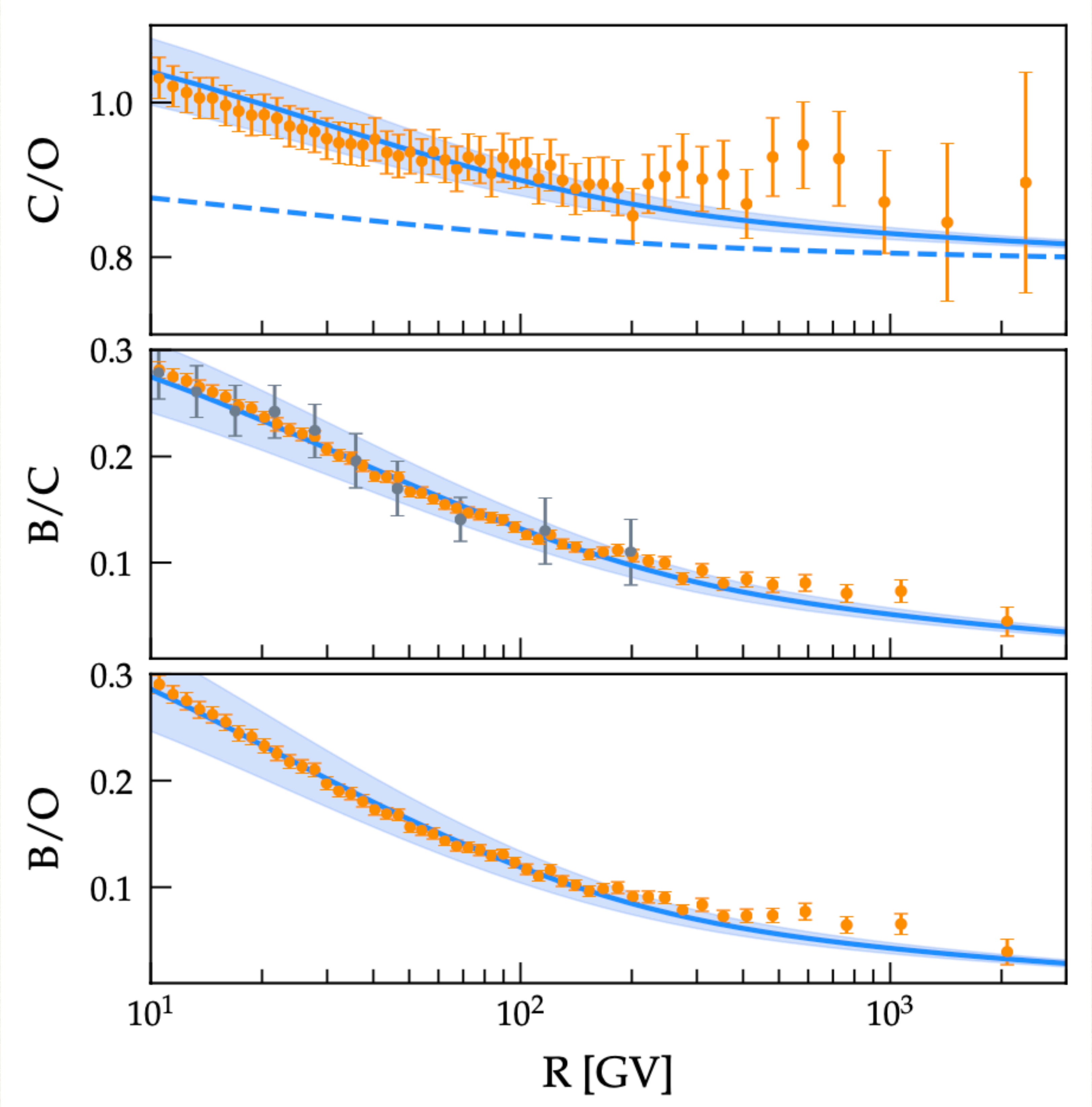
- Stationarity is ensured by proper boundary conditions $f_\alpha(z = \pm H) = 0$
- Diffusion $D(p)$ ← to be determined
- Advection by Galactic wind outflow mainly due to Alfvén speed: $u = u_w + v_A \sim v_A$
- Source term proportional to Galactic SN rate: $q_{\rm SN}(p) \propto \frac{\mathcal{R}E_{\rm SN}}{\pi R_{\rm disk}^2} f_{\rm esc}(p)$ ← to be determined
- Energy losses (ionization, Coulomb losses, IC, Synchrotron, …)
- production/destruction of nuclei due to inelastic scattering or decay $\rightarrow b_{\alpha'\alpha}\sigma_\alpha^{\rm in}$

Secondary-over-primary ratio

Evoli et al., PRD (2019); Weinrich et al. A&A 639 2020)

The model is applied to CR Nuclei to determine *D*/*H*





- A phenomenological motivated expression for the diffusion coefficient allow to fit all primary and secondary nuclei

$$\frac{D(R)}{H} = 2v_A + \frac{D_0}{H} \frac{\beta \, (R/\mathrm{GV})^{\delta}}{\left[1 + \left(R/R_b\right)^{\Delta s/s}\right]^{s}}$$

The presence of Alfvén speed and break are due to self generated turbulence

- However also in this case only D/H can be constrained

▷ by fitting primary and secondary/primary measurements we found:
$\delta \sim 0.54$, $D_0/H \sim 0.5 \times 10^{28}$ cm/s^2/kpc, $\Delta\delta \sim 0.2$, $v_A \sim 5$ km/s

Now we need to estimate *H*

Evolution of energy inside the SNR

Radiative losses

$$\left(\frac{dE}{dt}\right)_{\rm syn+IC} = -\frac{\sigma_{\rm T} c}{6\pi}\left(\frac{E}{m_e c^2}\right)^2\left(B^2 + B_{\rm eq}^2\right)$$

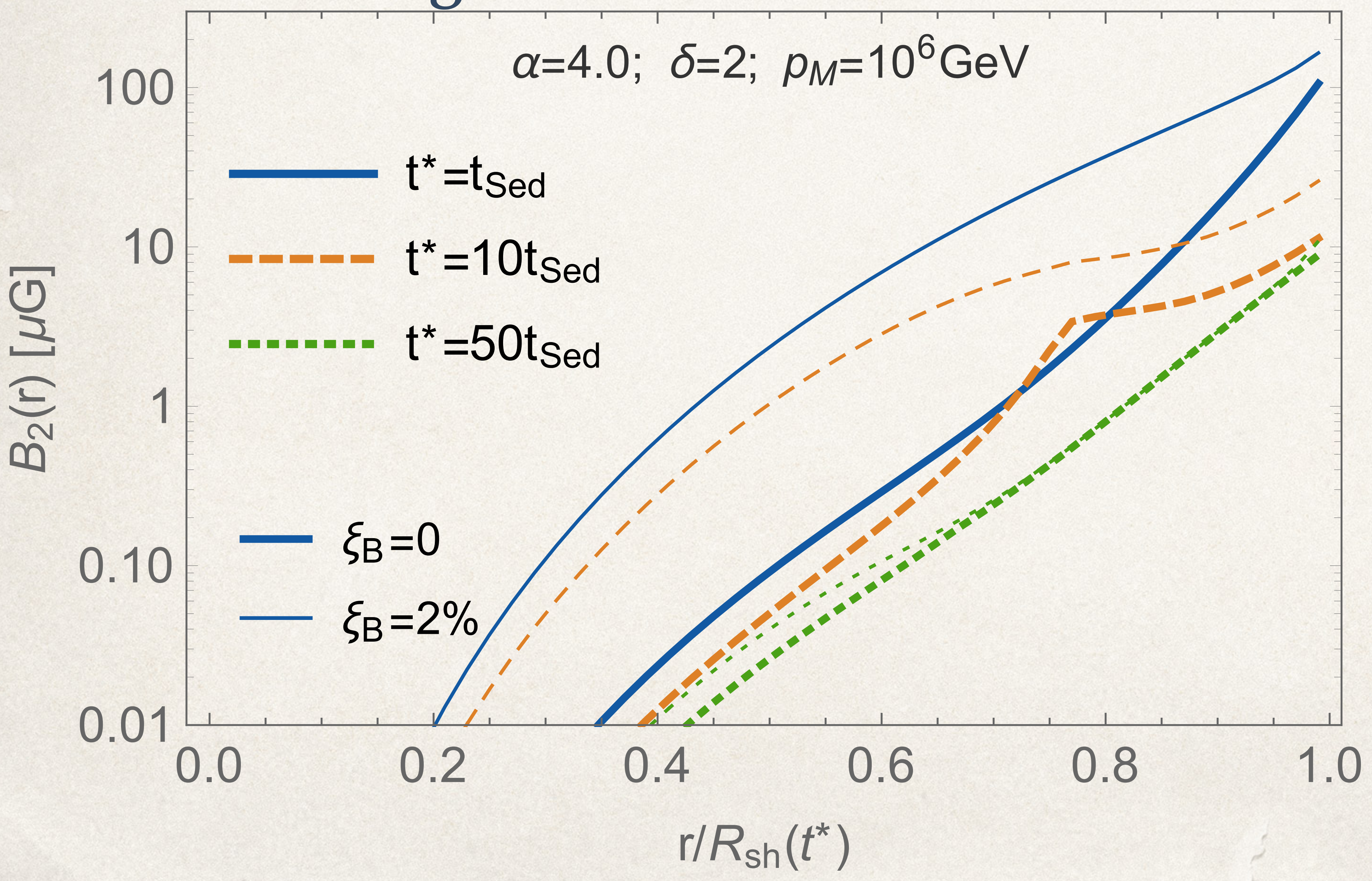
Adiabatic losses

$$\frac{dV'}{dV} = L^3(t',t)\,.$$

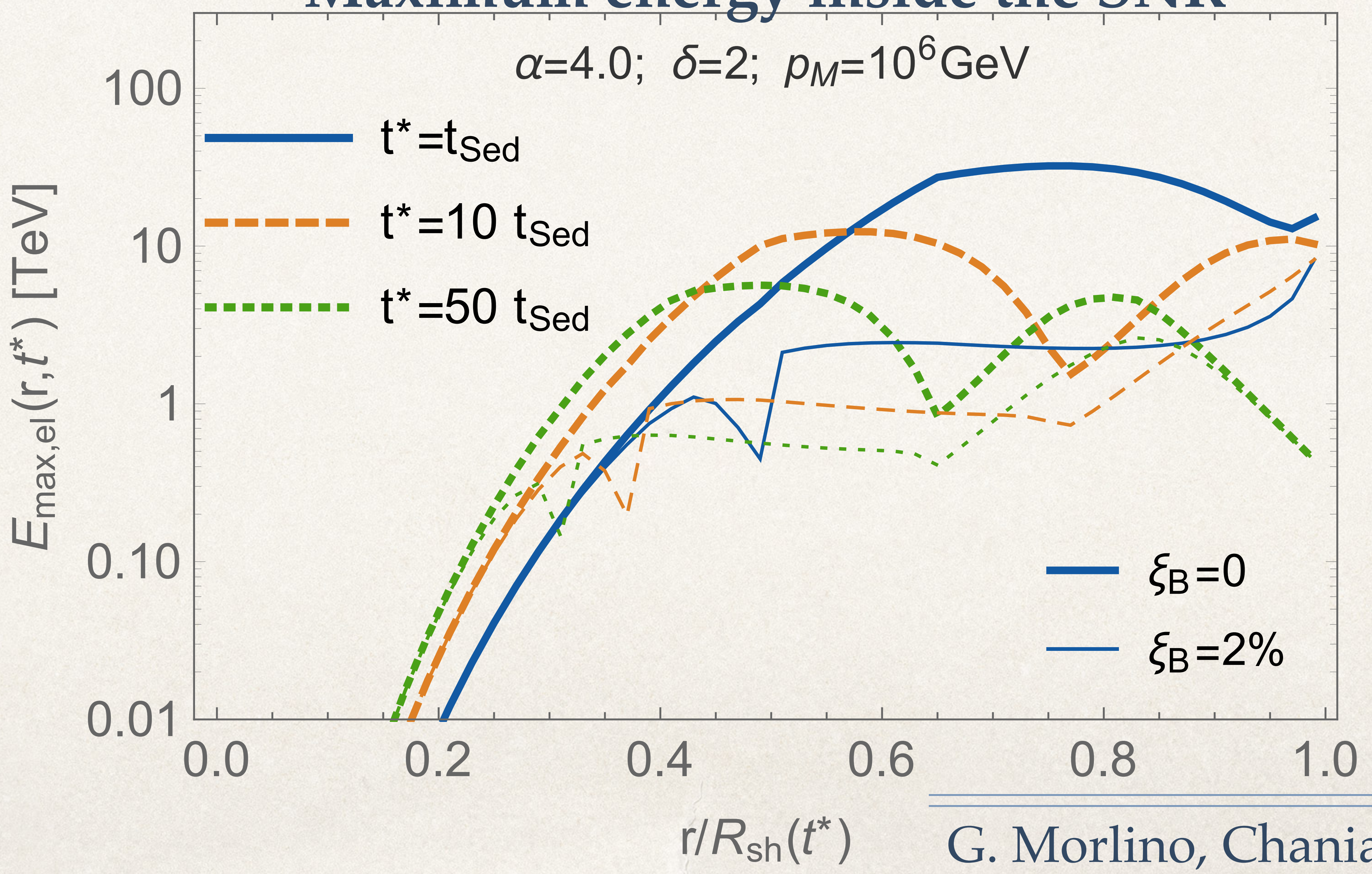
Total losses

$$\frac{dE}{dt} = \left(\frac{dE}{dt}\right)_{\rm syn+IC} + \frac{E}{L}\frac{dL}{dt},$$

Magnetic field inside the SNR



Maximum energy inside the SNR



Evolution of distribution function inside the SNR

Distribution function at the shock.

Protons:

$$f_0(p,t) = \frac{3\,\xi_{\mathrm{CR,p}}\, u_{\mathrm{sh}}^2(t)\rho_0}{4\pi\, c(m_p c)^4 \Lambda(p_{\mathrm{max},0}(t))} \left(\frac{p}{m_p c}\right)^{-\alpha} \exp\left[-\frac{p}{p_{\mathrm{max},0}(t)}\right]$$

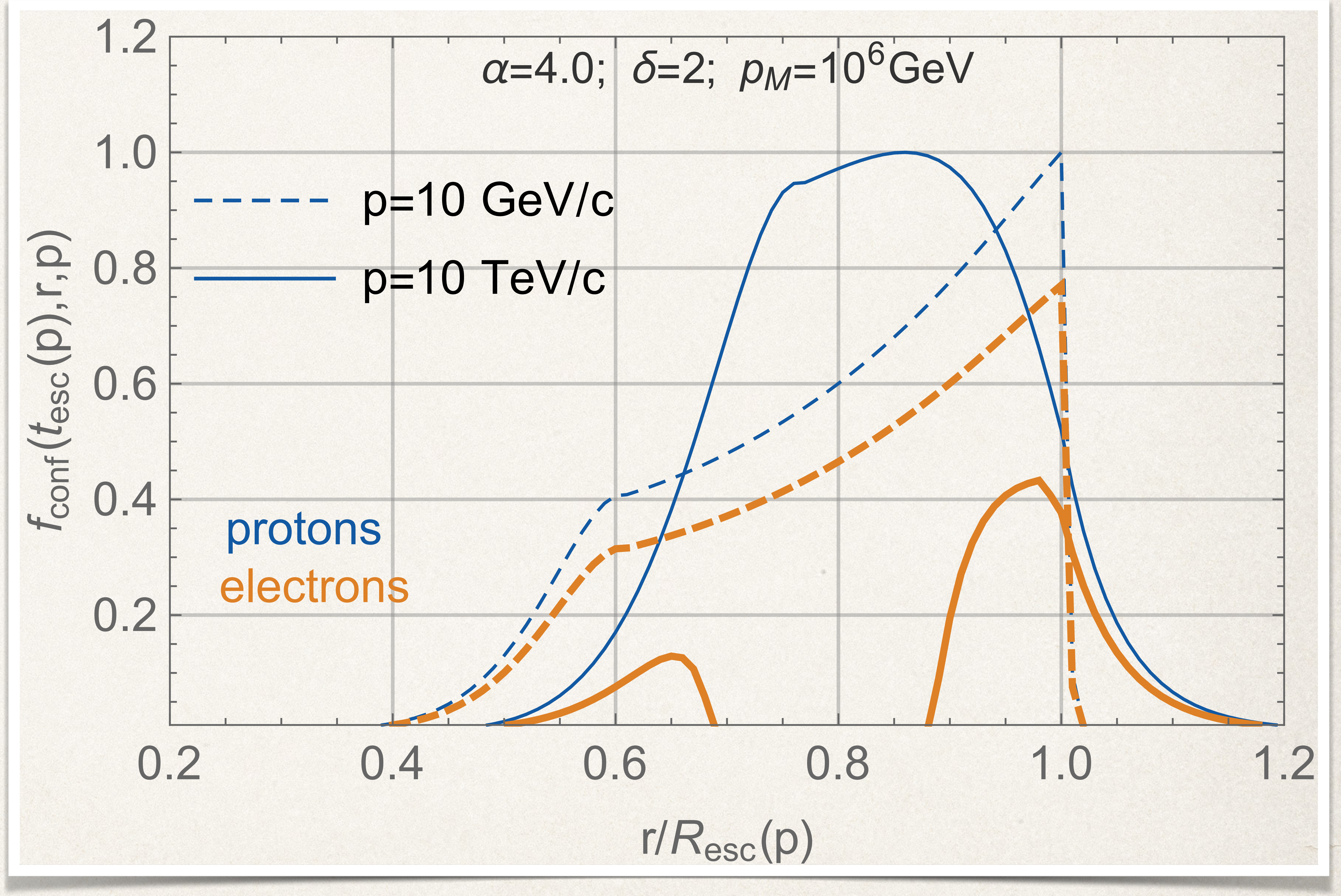
Electrons

$$f_{e,0}(p) = K_{\mathrm{ep}}\, f_{p,0}(p)\, e^{-p/p_{\mathrm{max}\,,loss}}$$

Time-evolution of distribution function

$$f_{e,\mathrm{conf}}(E,r,t) = f_{e,0}\left(\frac{E}{L(t',t) - IE}, t'\right) \frac{L}{(L-IE)^2} \frac{dV'}{dV}\,.$$

Distribution function inside the SNR at $t = t_{\mathrm{esc}}(p)$



SNR evolution (a very simple model…)

We account only for type-Ia SNe.

Expansion following Truelove & McKee(1999) in uniform ISM

$$\frac{R_{\rm sh}(t)}{R_{\rm ch}} = \begin{cases} 2.01\left(\frac{t}{t_{\rm ch}}\right)\left[1+1.72\left(\frac{t}{t_{\rm ch}}\right)^{3/2}\right]^{-2/3} & t < t_{\rm Sed} \\ \left[1.42\left(\frac{t}{t_{\rm ch}}\right)-0.254\right]^{2/5} & t \geq t_{\rm Sed} \end{cases}$$

$$\frac{u_{\rm sh}(t)}{u_{\rm ch}} = \begin{cases} 2.01\left[1+1.72\left(\frac{t}{t_{\rm ch}}\right)^{3/2}\right]^{-5/3} & t < t_{\rm Sed} \\ 0.569\left[1.42\left(\frac{t}{t_{\rm ch}}\right)-0.254\right]^{-3/5} & t \geq t_{\rm Sed} \end{cases}$$

Assumption:
acceleration stops at beginning of the radiative phase [Cioffi et al.(1988)]

$$t_{\rm SP} = 1.33\times 10^4\, E_{51}^{3/14} n_0^{-4/7} \zeta_m^{-5/14}\ {\rm yr}\,,$$

But why? Mach number at $t_{\rm SP}$ is still $\gtrsim 10$

Approximation for downstream velocity profile to estimate adiabatic losses [Ptuskin & Zirakashvili (2005)]

$$u(t,r) = \left(1-\frac{1}{\sigma}\right)\frac{u_{\rm sh}(t)}{R_{\rm sh}(t)}r\,,$$

Maximum energy and amplified magnetic field

We assume an arbitrary maximum energy and we derive the corresponding self-generated magnetic field

Protons maximum energy at the shock

$$p_{\max,0}(t) = \begin{cases} p_{\mathrm{M}}\left(t/t_{\mathrm{Sed}}\right) & \text{if } t < t_{\mathrm{Sed}} \\ p_{\mathrm{M}}\left(t/t_{\mathrm{Sed}}\right)^{-\delta} & \text{if } t < t_{\mathrm{Sed}} \end{cases}$$

$$t_{\mathrm{SNR}} = t_{\mathrm{acc}}(p_{\max,0}) = 8\frac{D_1(p_{\max})}{u_{\mathrm{sh}}^2}$$

$$D_1 = D_B/\mathcal{F}$$

Self-generated turbulence

$$\mathcal{F}(t) = \frac{8p_{\mathrm{M}}c}{3eB_0 c t_{\mathrm{Sed}}}\frac{u_{\mathrm{sh}}}{c}\begin{cases} 1 & \text{if } t < t_{\mathrm{Sed}} \\ \left(t/t_{\mathrm{Sed}}\right)^{-\delta-1} & \text{if } t > t_{\mathrm{Sed}} \end{cases}$$

$$\mathcal{F} = \begin{cases} (\delta B_1/B_0)^2 & \text{if } \delta B_1 \ll B_0 \\ \delta B_1/B_0 & \text{if } \delta B_1 \gg B_0 \end{cases}$$

Upstream magnetic field

$$\delta B_1(t) = \frac{B_0}{2}\left(\mathcal{F}(t) + \sqrt{4\mathcal{F}(t) + \mathcal{F}(t)^2}\right)$$

Electron maximum energy determined from losses due to δB_1

$$t_{\mathrm{acc}}\left(p_{\max,e}\right) = \min[\tau_{\mathrm{loss}}(\delta B_1), t_{\mathrm{SNR}}]$$