

Institute of
Space Sciences



Pulsar wind nebulae phenomenology at and beyond reverberation

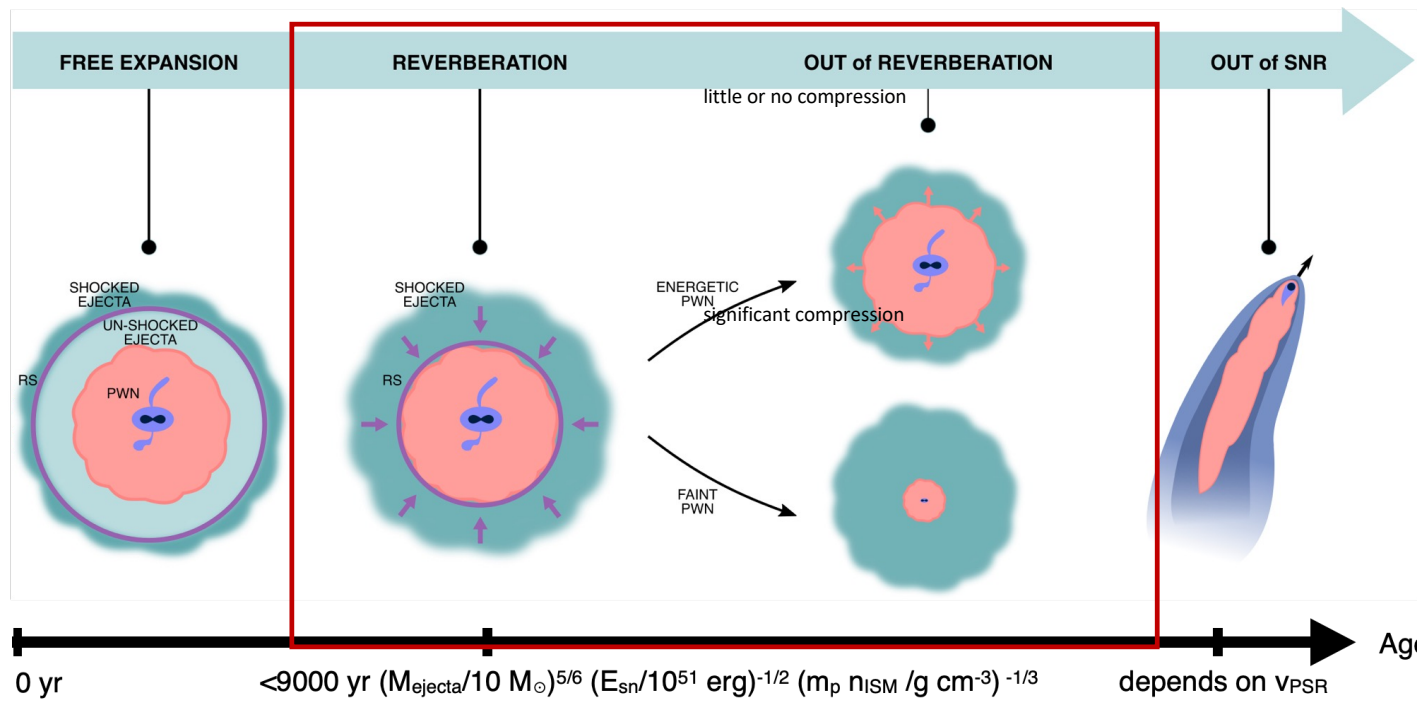
Diego F. Torres

Collaborators:

R. Bandiera, N. Bucciantini, B. Olmi (INAF), J. Martin (INAF, ICE-CSIC)

Reverberation phase, the great unknown

Plot from Olmi & Bucciantini 2023



Free expansion proceeds until the PWN reaches the reverse shock (RS).

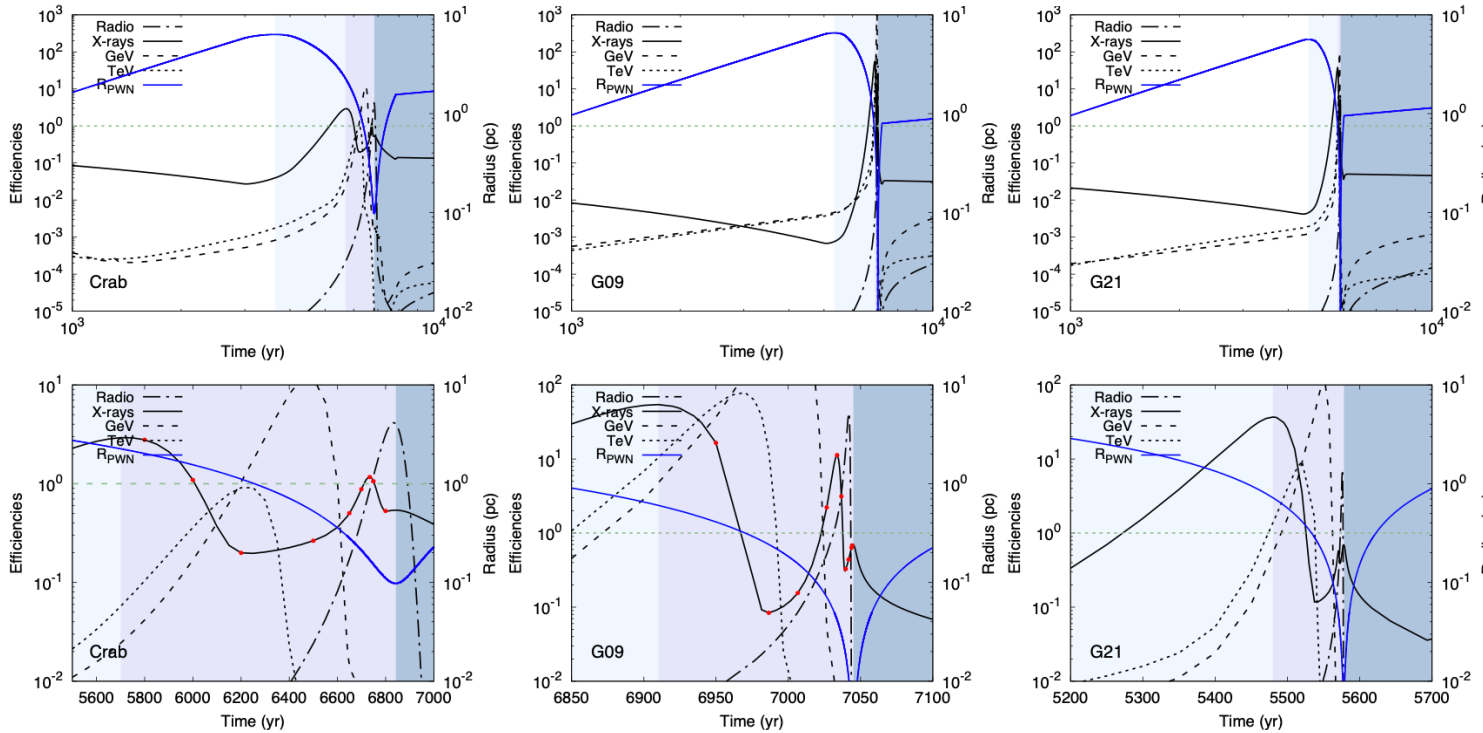
After that time, the shell experiences a strong deceleration, which in many cases leads to a compression of the PWN.

When due to compression the PWN internal pressure becomes high enough, the PWN bounces and re-expands again.

Reverberation (compression - bounce) last for a few kyrs or less, and happens relatively shortly after birth - at the end of free expansion.

Reverberation may produce a significant re-distribution of particle energies

DFT & Lin, ApJ Letters 2018; also; DFT, ApJ 2017; DFT, Lin & Coti-Zelati MNRAS 2019



The population features of all middle-age PWN depend on understanding reverberation.

- During reverberation the energy reservoir is no longer the pulsar spin-down power

- The environment gives energy to the nebula due to its compression

- Such compression will significantly increase the magnetic field, and thus the losses of particles.

- The particle population may be drastically changed, burnt, or have their energies reshuffled.

- PWN could be super-efficient: e.g., $L_x > L_{sd}$ at the time

Then, knowing the shock positions is key

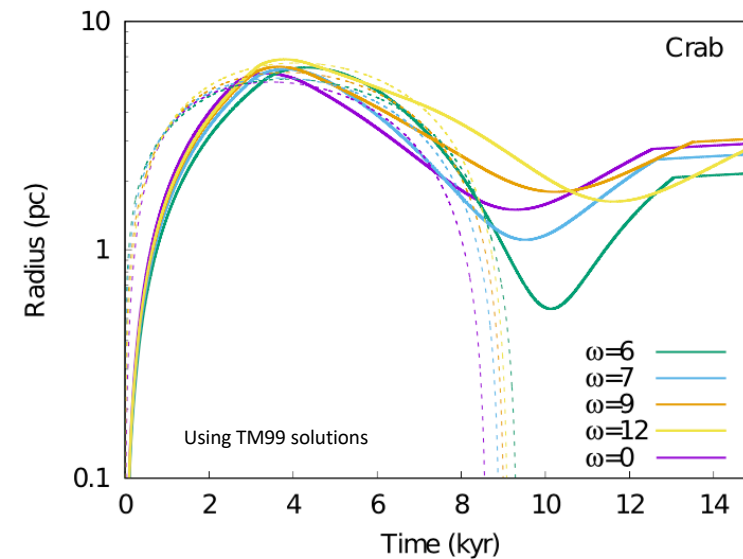
Bandiera, Bucciantini, Martin, Olmi & DFT, MNRAS 2020

Institute of
Space Sciences



Understanding the SNR evolution, particularly the position of the Reverse Shock, is fundamental when modelling the complex dynamics of the interaction between the SNR and its host PWN.

Even small variations of the position and velocity of the RS at the onset of reverberation may produce large variations in the final compression factor of the PWN.



Bandiera, Bucciantini, Martin, Olmi & DFT, MNRAS 2020

presents an in-depth study on what are the parameters affecting the most the amount of the compression at the onset of reverberation

Under the carpet... the shock positions

Bandiera, Bucciantini, Martin, Olmi & DFT, MNRAS 2021

Truelove and McKee (1999, TM99) solutions were used in all models when incorporating the dynamics in radiative models of pulsar wind nebulae (PWNe).

Through a mix of analytical limits, semi-analytical formulae, and fits to numerical simulations, TM99 provides a series of approximations to describe the evolution of the SNR during its different stages.

TM99 has become a widely used reference for the time evolution of the FS and RS.

Scalings

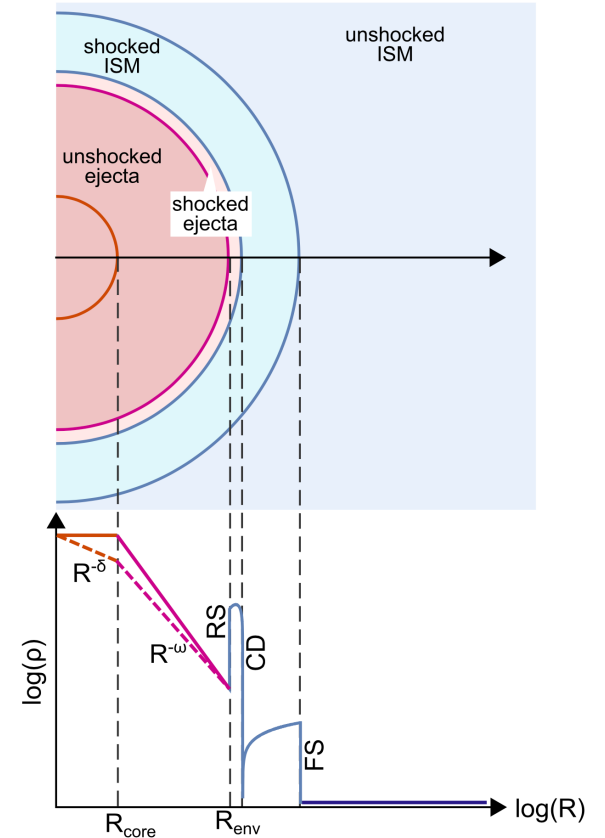
$$R_{\text{ch}} = M_{\text{ej}}^{1/3} \rho_0^{-1/3} \simeq 7.4 \text{ pc} \left(\frac{M_{\text{ej}}}{10 M_{\odot}} \right)^{1/3} \left(\frac{m_{\text{p}} n_0}{\text{g cm}^{-3}} \right)^{-1/3},$$

$$t_{\text{ch}} = E_{\text{sn}}^{-1/2} M_{\text{ej}}^{5/6} \rho_0^{-1/3} \simeq 3241 \text{ yr} \left(\frac{E_{\text{sn}}}{10^{51} \text{ erg}} \right)^{-1/2} \left(\frac{M_{\text{ej}}}{10 M_{\odot}} \right)^{5/6} \left(\frac{m_{\text{p}} n_0}{\text{g cm}^{-3}} \right)^{-1/3}$$

$$V_{\text{ch}} = \frac{E_{\text{sn}}^{1/2}}{M_{\text{ej}}^{1/2}} \simeq 2240 \text{ km s}^{-1} \left(\frac{E_{\text{sn}}}{10^{51} \text{ erg}} \right)^{1/2} \left(\frac{M_{\text{ej}}}{10 M_{\odot}} \right)^{-1/2}$$

Typical models assume a core with a shallow radial profile $\propto r^{-\delta}$ (with $\delta < 3$), plus an envelope with a steep power-law one $\propto r^{-\omega}$ (with $\omega > 5$),

$$\rho_{\text{ej}}(r, t) = \begin{cases} A (v_t/r)^{\delta} / t^{3-\delta}, & \text{if } r < v_t t, \\ A (v_t/r)^{\omega} t^{\omega-3}, & \text{if } v_t t \leq r < R_{\text{RS}} \end{cases} \quad \begin{cases} v_t = \sqrt{\frac{2(5-\delta)(\omega-5)}{(3-\delta)(\omega-3)} \frac{E_{\text{sn}}}{M_{\text{ej}}}} \\ A = \frac{(5-\delta)(\omega-5)}{2\pi(\omega-\delta)} \frac{E_{\text{sn}}}{v_t^5} \end{cases}$$



Then, knowing the shock positions is key

Bandiera, Bucciantini, Martin, Olmi & DFT, MNRAS 2020

Institute of
Space Sciences



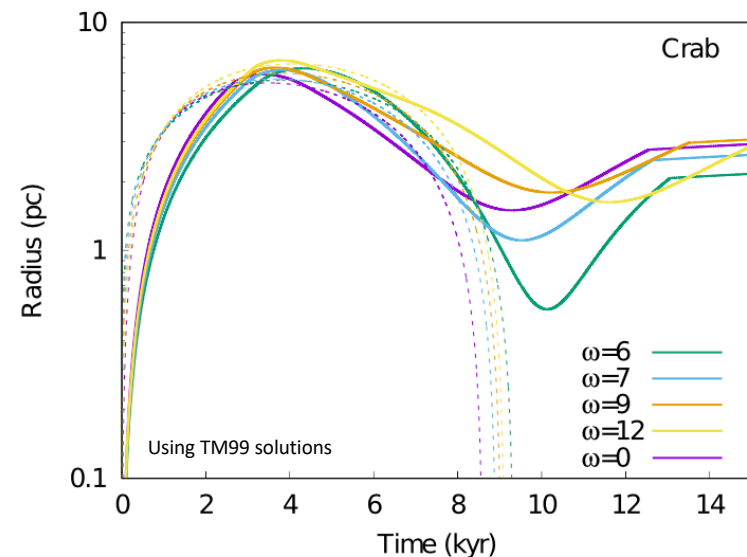
Understanding the SNR evolution, particularly the position of the Reverse Shock, is fundamental when modelling the complex dynamics of the interaction between the SNR and its host PWN

Even small variations of the position and velocity of the RS at the onset of reverberation may produce large variations in the final compression factor of the PWN

Note that according to ω (the power law index of the ejecta profile) the time and extent of the compression varies a lot.

Not even linearly! (which already casts doubts on the TM99 solutions)

We really need a better prescription



Shock positions

• Direct comparison of our formulae for the characteristic trajectories RS, CD, FS with those of [TM99](#) (only valid for $\delta = 0$).

Truelove J. K., McKee C. F., 1999, ApJS, 120, 299 (TM99)		ω	t	SUPPORTING FORMULAS
	$1.83t(1 + 3.26t^{3/2})^{-2/3}$	$\omega = 0$	$t < t_{st}$	$t_{st} = (2\omega/[5(\omega - 3)]\hat{A}^{-\omega/(\omega-3)}\sqrt{2.026})^{2\omega/[3(\omega-5)]}$
	$t(0.779 - 0.106t - 0.533\ln t)$	$\omega = 0$	$t \geq t_{st}$	$\hat{A} = \{27l_{ed}^{(\omega-2)}/[4\pi\omega(\omega-3)\phi_{ed}](10[\omega-5]/[3(\omega-3)])^{(\omega-3)/2}\}^{1/\omega}$
				$l_{ed} = \{1.39, 1.26, 1.21, 1.19, 1.17, 1.15, 1.14\}^*$
RS	$\hat{A}t^{(\omega-3)/\omega}$	$5 < \omega \leq 14$	$t \leq t_{core}$	$t_{core} = [27/(4\pi\omega[\omega-3]l_{ed}^2\phi_{ed})]^{1/3}\sqrt{3(\omega-3)/(10[\omega-5])}$
	$t(\hat{A}t_{core}^{(\omega-3)/\omega}/l_{ed})$	$5 < \omega \leq 14$	$t_{core} < t \leq t_{st}$	$\phi_{ed} = \{0.39, 0.47, 0.52, 0.55, 0.57, 0.6, 0.62\}^*$
	$t[(\hat{A}t_{core}^{(\omega-3)/\omega})/(l_{ed}t_{core}) - a_{core}(t - t_{core}) +$ $-(3/(l_{ed}\omega)\hat{A}t_{core}^{[(\omega-3)/\omega]-1} - a_{core}t_{core})\ln(t/t_{core})]$	$5 < \omega \leq 14$	$t > t_{st}$	$a_{core} = \{0.112, 0.116, 0.139, 0.162, 0.192, 0.251, 0.277\}^*$
CD	—	—	—	—
FS	$2.01t(1 + 1.72t^{3/2})^{-2/3}$	$\omega = 0$	$t < t_{st}$	
	$(1.42t - 0.254)^{2/5}$	$\omega = 0$	$t \geq t_{st}$	
	$\hat{A}t^{(\omega-3)/\omega}$	$5 < \omega \leq 14$	$t \leq t_{st}$	
	$(\hat{A}t_{st}^{(\omega-3)/\omega})^{5/2} + \sqrt{2.026}(t - t_{st})^{2/5}$	$5 < \omega < 14$	$t > t_{st}$	

*All the listed numerical values must be considered relative to the range $\omega = \{6, 7, 8, 9, 10, 12, 14\}$.

Shock positions

• Direct comparison of our formulae for the characteristic trajectories RS, CD, FS with those of [TM99](#) (only valid for $\delta = 0$).

Truelove J. K., McKee C. F., 1999, ApJS, 120, 299 (TM99)		ω	t	SUPPORTING FORMULAS
	$1.83t(1 + 3.26t^{3/2})^{-2/3}$	$\omega = 0$	$t < t_{st}$	$t_{st} = (2\omega/[5(\omega - 3)]\hat{A}^{-\omega/(\omega-3)}\sqrt{2.026})^{2\omega/[3(\omega-5)]}$
	$t(0.779 - 0.106t - 0.533\ln t)$	$\omega = 0$	$t \geq t_{st}$	$\hat{A} = \{27l_{ed}^{(\omega-2)}/[4\pi\omega(\omega-3)\phi_{ed}][10[\omega-5]/[3(\omega-3)]]^{(\omega-3)/2}\}^{1/\omega}$
				$l_{ed} = \{1.39, 1.26, 1.21, 1.19, 1.17, 1.15, 1.14\}^*$
RS	$\hat{A}t^{(\omega-3)/\omega}$	$5 < \omega \leq 14$	$t \leq t_{core}$	$t_{core} = [27/(4\pi\omega[\omega-3]l_{ed}^2\phi_{ed})]^{1/3}\sqrt{3(\omega-3)/(10[\omega-5])}$
	$t(\hat{A}t_{core}^{(\omega-3)/\omega}/l_{ed})$	$5 < \omega \leq 14$	$t_{core} < t \leq t_{st}$	$\phi_{ed} = \{0.39, 0.47, 0.52, 0.55, 0.57, 0.6, 0.62\}^*$
	$t[(\hat{A}t_{core}^{(\omega-3)/\omega})/(l_{ed}t_{core}) - a_{core}(t - t_{core}) +$ $-(3/(l_{ed}\omega)\hat{A}t_{core}^{[(\omega-3)/\omega]-1} - a_{core}t_{core})\ln(t/t_{core})]$	$5 < \omega \leq 14$	$t > t_{st}$	$a_{core} = \{0.112, 0.116, 0.139, 0.162, 0.192, 0.251, 0.277\}^*$
CD	—	—	—	—
	$2.01t(1 + 1.72t^{3/2})^{-2/3}$	$\omega = 0$	$t < t_{st}$	
	$(1.42t - 0.254)^{2/5}$	$\omega = 0$	$t \geq t_{st}$	
FS	$\hat{A}t^{(\omega-3)/\omega}$	$5 < \omega \leq 14$	$t \leq t_{st}$	
	$(\hat{A}t_{st}^{(\omega-3)/\omega})^{5/2} + \sqrt{2.026}(t - t_{st})^{2/5}$	$5 < \omega < 14$	$t > t_{st}$	

We did 1D Lagrangian simulations for a large number of PWN, with different energetics, ω and δ parameters, and different characteristic values of the system and directly measured the shock positions.

Then fitted with functions providing much better than 1% accuracy, and compared the results with TM99 solutions.

*All the listed numerical values must be considered relative to the range $\omega = \{6, 7, 8, 9, 10, 12, 14\}$.

Shock positions

• Direct comparison of our formulae for the characteristic trajectories RS, CD, FS with those of [TM99](#) (only valid for $\delta = 0$).

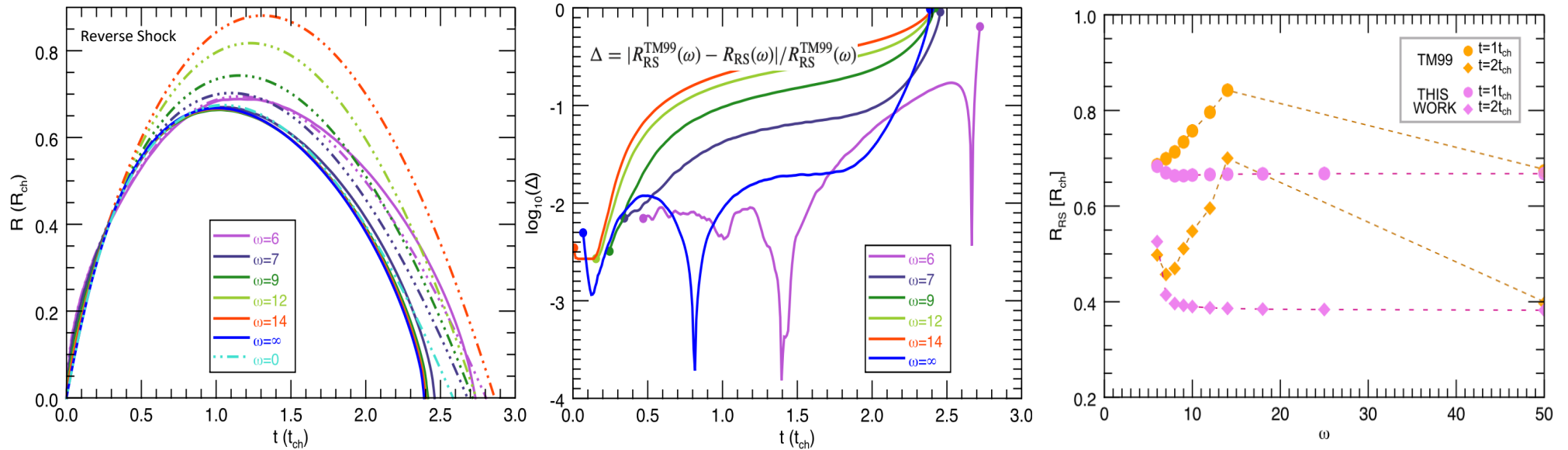
Truelove J. K., McKee C. F., 1999, ApJS, 120, 299 (TM99)		ω	t	SUPPORTING FORMULAS
	$1.83t(1 + 3.26t^{3/2})^{-2/3}$	$\omega = 0$	$t < t_{st}$	$t_{st} = (2\omega/[5(\omega - 3)]\hat{A}^{-\omega/(\omega-3)}\sqrt{2.026})^{2\omega/[3(\omega-5)]}$ $\hat{A} = \{27l_{ed}^{(\omega-2)}/[4\pi\omega(\omega-3)\phi_{ed}]\{10[\omega-5]/[3(\omega-3)]\}^{(\omega-3)/2}\}^{1/\omega}$ $l_{ed} = \{1.39, 1.26, 1.21, 1.19, 1.17, 1.15, 1.14\}^*$
	$t(0.779 - 0.106t - 0.533\ln t)$	$\omega = 0$	$t \geq t_{st}$	
RS	$\hat{A}t^{(\omega-3)/\omega}$ $t(\hat{A}t_{core}^{(\omega-3)/\omega}/l_{ed})$	$5 < \omega \leq 14$	$t \leq t_{core}$	$t_{core} = [27/(4\pi\omega[\omega-3]l_{ed}^2\phi_{ed})]^{1/3}\sqrt{3(\omega-3)/(10[\omega-5])}$ $\phi_{ed} = \{0.39, 0.47, 0.52, 0.55, 0.57, 0.6, 0.62\}^*$
	$t[(\hat{A}t_{core}^{(\omega-3)/\omega})/(l_{ed}t_{core}) - a_{core}(t - t_{core}) +$ $-(3/(l_{ed}\omega)\hat{A}t_{core}^{[(\omega-3)/\omega]-1} - a_{core}t_{core})\ln(t/t_{core})]$	$5 < \omega \leq 14$	$t > t_{st}$	
CD	—	—	—	—
	$2.01t(1 + 1.72t^{3/2})^{-2/3}$	$\omega = 0$	$t < t_{st}$	
	$(1.42t - 0.254)^{2/5}$	$\omega = 0$	$t \geq t_{st}$	
FS	$\hat{A}t^{(\omega-3)/\omega}$	$5 < \omega \leq 14$	$t \leq t_{st}$	
	$(\hat{A}t_{st}^{(\omega-3)/\omega})^{5/2} + \sqrt{2.026}(t - t_{st})^{2/5}$	$5 < \omega < 14$	$t > t_{st}$	
Bandiera, Bucciantini, Martin, Olmi & DFT, MNRAS 2021		ω	t	SUPPORTING FORMULAS
RS	$\mathcal{R}(x) \times \mathcal{F}(\omega)$	$\omega \geq 6$	$t_{core} < t \leq t_{implo}$	$x = t/t_{implo}(\omega, \delta)$ $t_{implo} = 2.399 + \{(0.1006\Omega)^2 +$ $+ [(-0.06494 + 0.7063\Omega)/(1 + 1/\Omega^2)]^2\}^{0.5}$ $\Omega = 1/(\omega - 5)$ $\mathcal{R}(x) = [x^{1.5551}(1-x)^{0.68236}] [0.01961 + 0.5093x + 0.1874x^2]^{-1}$ $\mathcal{F}(\omega) = 1 + \{0.02171[\Omega/0.3338 - 1]\}\{1 + [\Omega/0.3338]^{-2.778}\}^{-1}$
CD	$\tilde{a}(\omega)t^{(\omega-3)/\omega}/[1 + b_{CD}(\omega)t^{c_{CD}(\omega)}]$	$\omega \geq 6$	$t_{core,CD} < t \leq t_{implo}$	$\tilde{a}(\omega) = (1.141 + 1.806\omega)(7.636 + \omega)^{-1}$ $b_{CD} = -1.051 - 0.1961\tilde{a}(\omega)$ $c_{CD} = -(5.561 + 0.6741\omega)(\omega - 4.826)^{-1}$
FS	$\xi_0(t + 1.94)^{2/5}/[1 + 0.672(1/t) + 0.00373(1/t)^2]$	$\omega \geq 6$	$t > t_{core,FS}$	$\xi_0 = 1.15169$ (from the standard Sedov solution)

*All the listed numerical values must be considered relative to the range $\omega = \{6, 7, 8, 9, 10, 12, 14\}$.

Valid for all w larger than 6, all values of δ .

Shock positions

Bandiera, Bucciantini, Martin, Olmi & DFT, MNRAS 2021



**The relative variation between models is typically of order of 10 to 30% per cent.
Impact of a 10% change in the compression cannot be neglected!**

The loss of accuracy in the TM99 approximation leads to **even larger relative differences closer to the implosion time, of the order of 100 per cent!**

The TM99 solutions do not converge, for large ω values, to their asymptotic solution $\omega = 0$ ($\omega = \infty$ in our notation).

PWN 1-zone-models: literature either ignores reverberation at all, or treats it in simplified ways

In the latter case, there are 3 main assumptions:

1. The PWN is a **uniform** (one-zone) bubble of particles and field
2. The shell at the PWN boundary is **thin** ($R_{\text{shell}} \sim R_{\text{pwn}} ; \Delta R_{\text{shell}} \ll R_{\text{pwn}}$)
3. The **pressure outside** the PWN is **equal to or a constant fraction of** the pressure at the FS in the **Sedov** solution

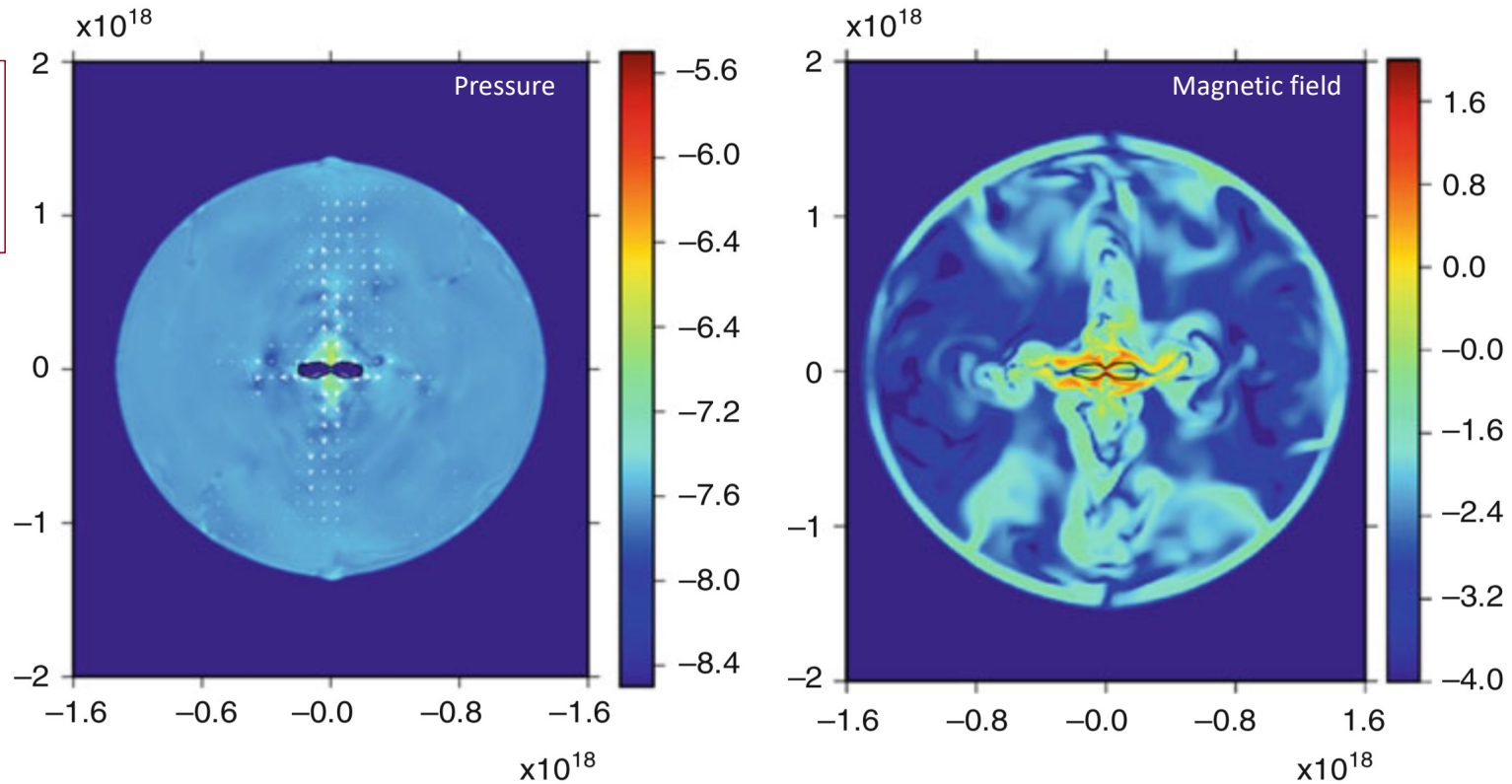
$$P_{\text{Sedov}} = 0.1592 \left(\frac{t}{t_{\text{ch}}} \right)^{-6/5} \frac{\rho_{\text{ISM}} E_{\text{sn}}}{M_{\text{ej}}}$$

[Gelfand et al. 2009 - Fang & Zhang 2010 - Tanaka & Takahara 2010 - Martin et al. 2012 - Tanaka & Takahara 2013 - Vorster et al. 2013 - Torres et al. 2013-2014- 2017-2018-2019 - Gelfand et a. 2015-2017 - Bandiera et al. 2021 - Fiori et al. 2022]

First assumption... roughly ok, according to 3D MHD models

Porth et al. 2014
Crab @ ~50 years

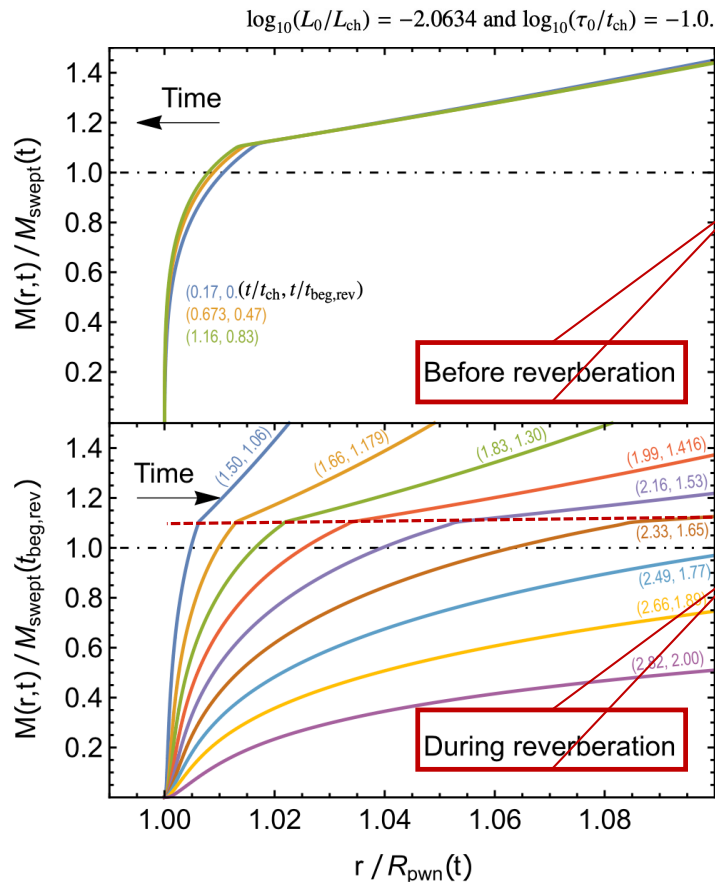
Olmi et al. 2019
Crab @ ~250 years



The PWN is a rather uniform (one-zone) bubble of particles and field.

The thin-shell is not thin in reverberation

Bandiera, Bucciantini, Martin, Olmi & DFT, MNRAS 2023a



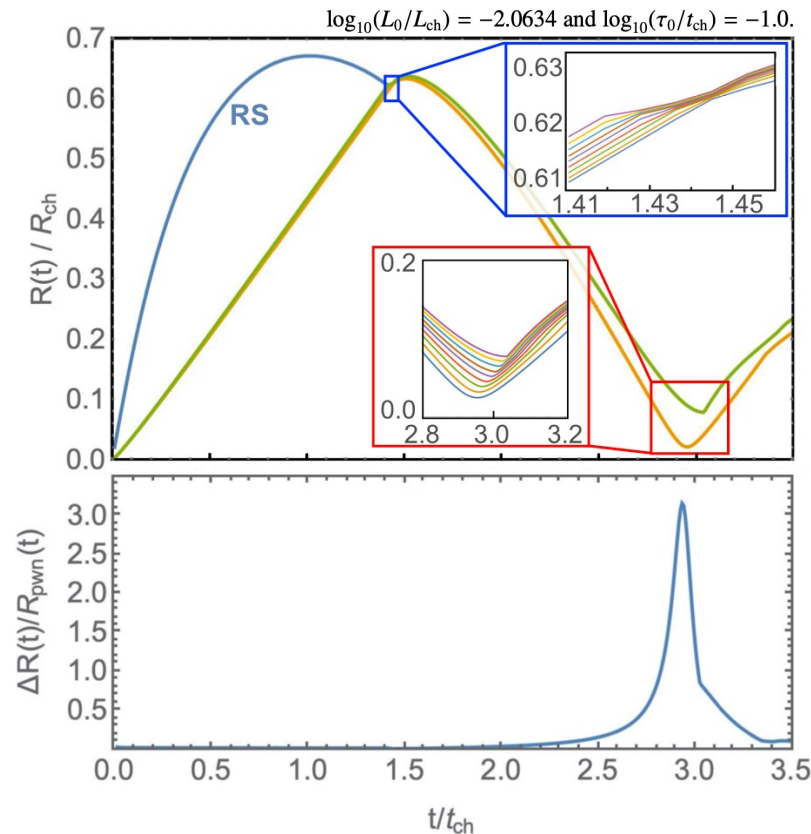
- For a scaled radius ranging between 1 and 1.02, the profiles are related to the density profile inside the shell.
- They are superimposed, meaning that the density preserves its profile, apart from a slight decrease with time of the shell relative width.
- The sharp break in the profiles (reflecting a density jump) indicates the position of the shock at the outer boundary of the shell;
- The scaled mass higher than unity means that the real swept-up mass is that within the outer boundary of the shell, rather than within $R_{\text{pwn}}(t)$.

- After $t_{\text{beg,rev}}$ the mass within the shell, now scaled with the swept-up mass at $t_{\text{beg,rev}}$, does not change with time: note the constancy of the vertical coordinate of the break
- The relative width of the shell increases with time, partly reflecting its physical broadening, and partly as a consequence PWN decreasing of its size.

This figure justifies the assumption of a **fixed shell mass during reverberation** and that when the PWN has been compressed, the **needed conditions for treating the shell as a thin-shell are no longer valid.**

The thin-shell is not thin in reverberation

Bandiera, Bucciantini, Martin, Olmi & DFT, MNRAS 2023a



For $t < t_{\text{beg,rev}}$ the shell boundaries are very close each other, meaning that the thin-shell approximation is well satisfied.

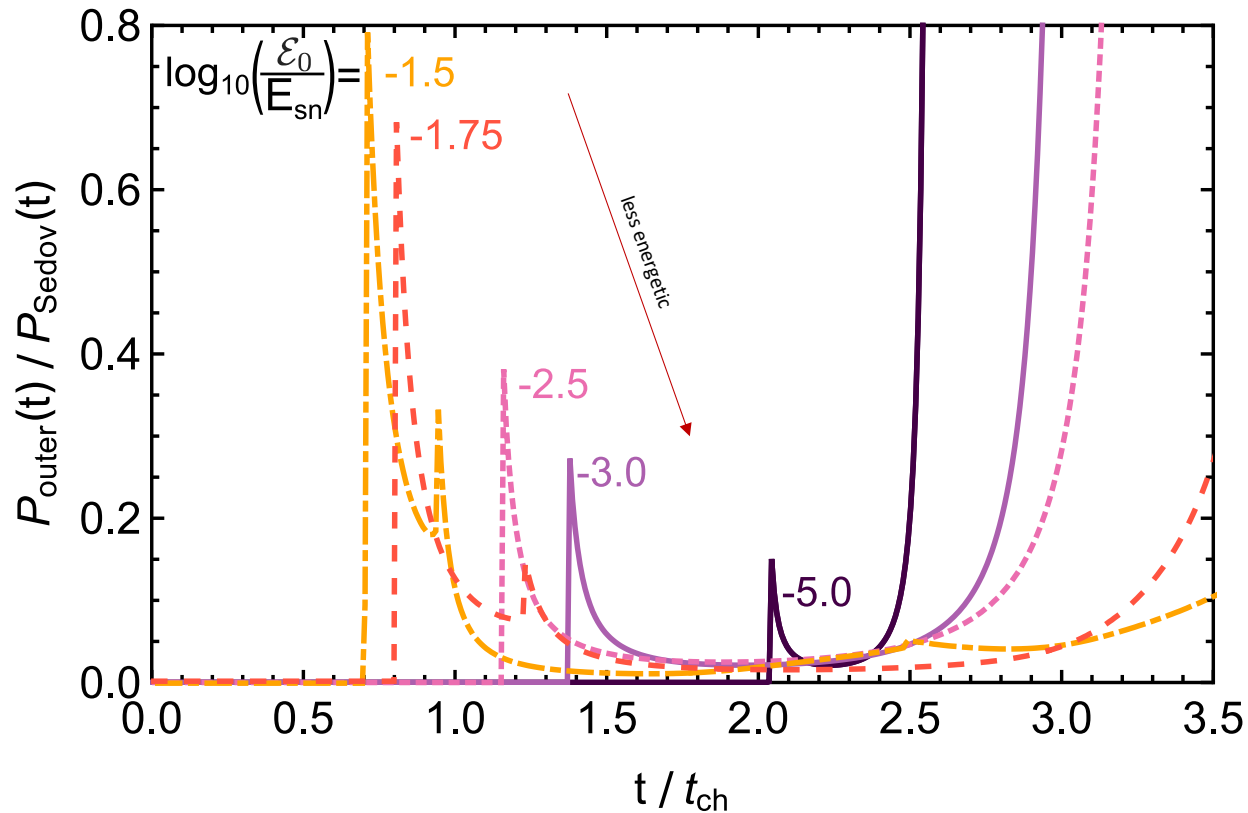
But especially close to the maximum compression, the shell boundaries separate, and the combination of a higher shell thickness and a smaller shell size implies that a thin-shell approach is no longer justified.

During the reverberation phase, the outer edge of the shell is defined by the mass collected before $t_{\text{beg,rev}}$ and one may clearly see that the shell becomes thicker, and as the PWN starts to contract the shell inflates progressively

The outer pressure is not a constant nor Sedov like

Bandiera, Bucciantini, Martin, Olmi & DFT, MNRAS 2023a

Institute of
Space Sciences



$\mathcal{E}_0 = L_0 \tau_0$ is the PWN energetics

$$P_{\text{Sedov}}(t) = 0.0489 \left(\frac{t}{t_{\text{ch}}}\right)^{-6/5} \frac{\rho_0 E_{\text{sn}}}{M_{\text{ej}}}$$

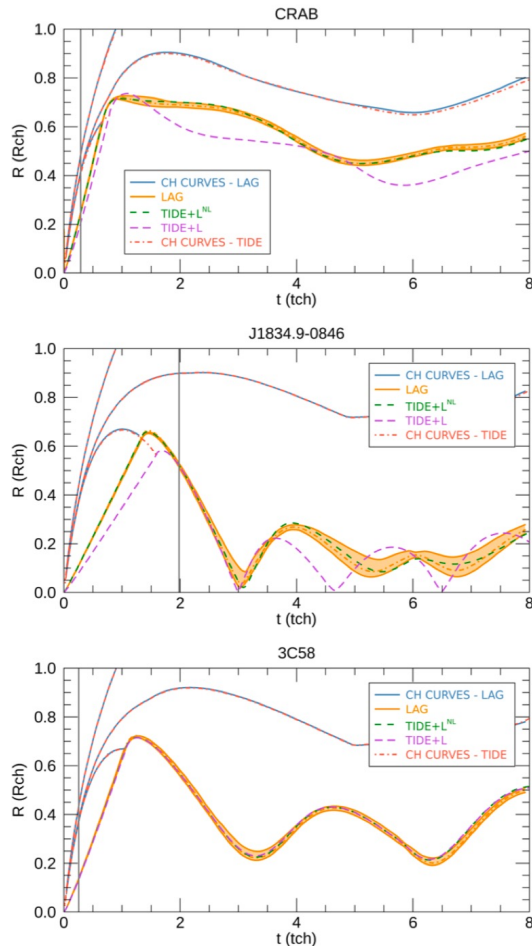
For a large part of the evolution P_{outer} is smaller than the Sedov pressure, and is different from a constant.

How to model middle age systems?

Bandiera, Bucciantini, Martin, Olmi & DFT, MNRAS 2023b

TIDE+L

Hybrid radiative – HD model



- Radiative model until reverberation, with dynamics incorporated
 - then use this stage just before reverberation as input for a...
- Lagrangian model thereafter,
 - But with radiation incorporated
- Correctly converges to the Lagrangian model all along when no losses are considered.
- Correctly matches at the interface between the two approaches.
- Relative fast for reasonable computational grid (few minutes per PWN).
- Can go to whatever age in a safe manner.

(State-of-the-art Radiative PWN code: correct shock positions, radiative, full ODEs, time-energy-dependent, Lagrangian after Reverberation)

Of course, recall, all of this is 3D in nature. Still an approximation.

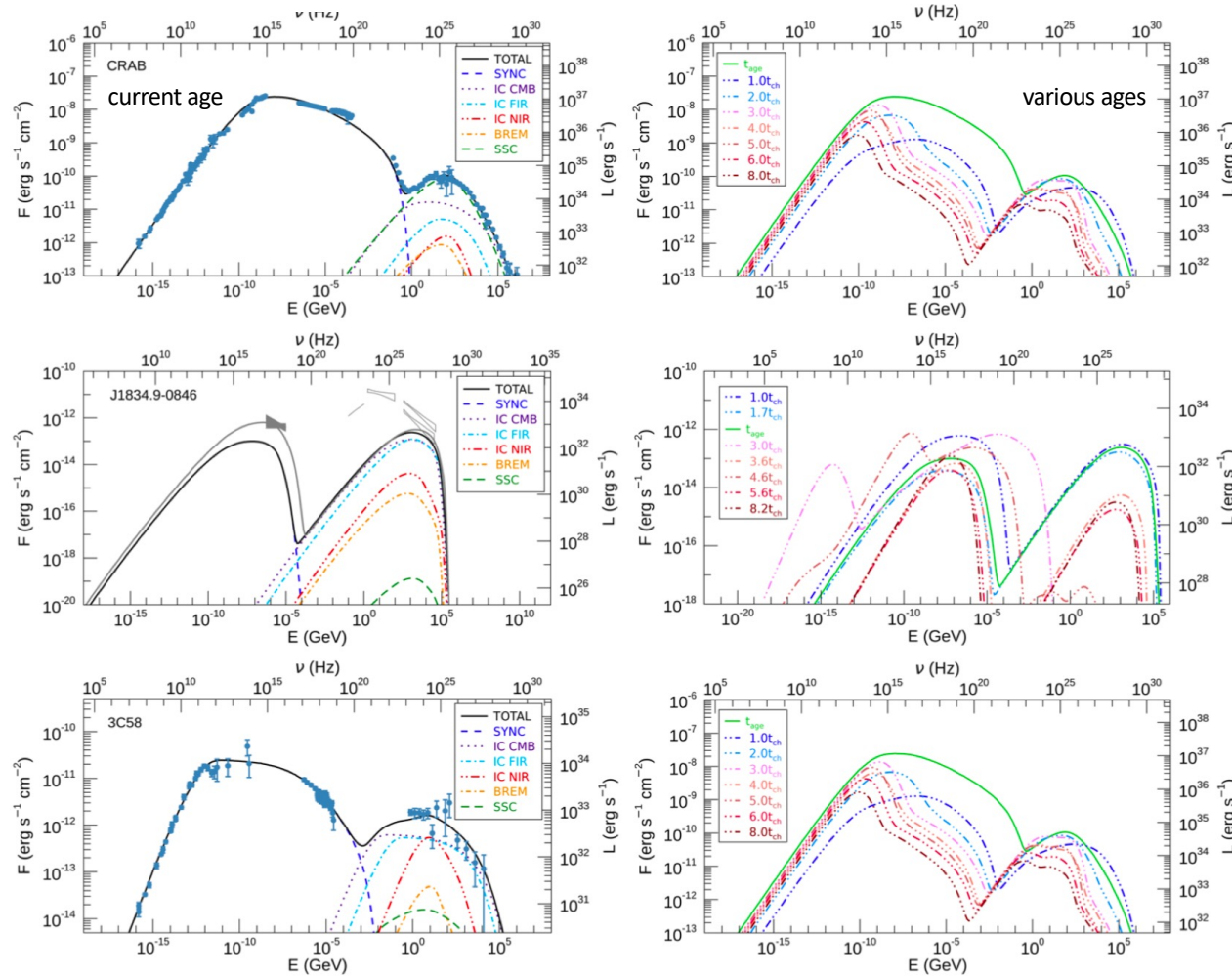
How to model middle age systems? Spectra

Bandiera, Bucciantini, Martin, Olmi & DFT, MNRAS 2023b

Significant spectral variability as time goes by

Superefficiency

Population analysis finally possible



To know more details see our recent series of papers

Institute of
Space Sciences



Bandiera, Bucciantini, Martin, Olmi & DFT, MNRAS 2021

Revisiting the evolution of non-radiative supernova remnants: a hydrodynamical-informed parametrization of the shock positions [Get access >](#)

R Bandiera , N Bucciantini, J Martín, B Olmi , D F Torres

Monthly Notices of the Royal Astronomical Society, Volume 508, Issue 3, December 2021, Pages 3194–3207, <https://doi.org/10.1093/mnras/stab2600>

Published: 13 September 2021 **Article history** ▾

Bandiera, Bucciantini, Martin, Olmi & DFT, MNRAS 2020

Reverberation of pulsar wind nebulae (I): impact of the medium properties and other parameters upon the extent of the compression [Get access >](#)

R Bandiera , N Bucciantini, J Martín , B Olmi, D F Torres 

Monthly Notices of the Royal Astronomical Society, Volume 499, Issue 2, December 2020, Pages 2051–2062, <https://doi.org/10.1093/mnras/staa2956>

Published: 28 September 2020 **Article history** ▾

Bandiera, Bucciantini, Martin, Olmi & DFT, MNRAS 2023a

Reverberation of pulsar wind nebulae – II. Anatomy of the ‘thin-shell’ evolution [Get access >](#)

R Bandiera , N Bucciantini , J Martín, B Olmi , D F Torres

Monthly Notices of the Royal Astronomical Society, Volume 520, Issue 2, April 2023, Pages 2451–2472, <https://doi.org/10.1093/mnras/stad134>

Published: 14 January 2023 **Article history** ▾

Bandiera, Bucciantini, Martin, Olmi & DFT, MNRAS 2023a

Reverberation of pulsar wind nebulae – III. Modelling of the plasma interface empowering a long term radiative evolution [Get access >](#)

R Bandiera , N Bucciantini , B Olmi, D F Torres

Monthly Notices of the Royal Astronomical Society, Volume 525, Issue 2, October 2023, Pages 2839–2850, <https://doi.org/10.1093/mnras/stad2387>

Published: 04 August 2023 **Article history** ▾

Conclusions

- Go beyond Truelove and McKee, we now have well-behaving formulae mimicking directly HD simulations.
- Mixed one-zone/HD models solve the problems introduced by the pure thin-shell approximation in the treatment of the reverberation phase: they catch the global behavior and estimate the PWN compression
- First relatively consistent numerical passage through reverberation for a radiative/HD model
- Some PWNe can reduce themselves in size by more than two orders of magnitude, at least in the 1D representation.
- Such systems with large compression $CF \gg 100$ are possible but rare, limited to the extremes of the known population of PWNe.
- Reductions in size by a factor of several up to an order of magnitude are quite common.
- As a result, superefficiency appears often at UV/optical, and less often in X-rays
- Understanding and correctly modelling reverberation is critical for population studies (e.g., how many PWN will we see in future surveys at different frequencies) and individual predictions / description of all middle-age systems

Institute of
Space Sciences



Thank you

<https://sites.google.com/view/dft-research>

@dft_research



Institute of
Space Sciences

